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COMPUTER INVESTIGATION OF LOW ENERGY NEUTRON-NEUTRON SCATTERING

FRANKLIN B. OSGOOD

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COMPUTER INVESTIGATION
OF LOW ENERGY
NEUTRON-NEUTRON SCATTERING

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Franklin B. Osgood

COMPUTER INVESTIGATION
OF LOW ENERGY
NEUTRON-NEUTRON SCATTERING

by

Franklin B. Osgood

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

IN

PHYSICS

United States Naval Postgraduate School

Monterey, California

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ABSTRACT

A proposed neutron-neutron scattering experiment in which the neutron source would be colliding beams from an underground nuclear explosion, was investigated. In particular, the accuracy to be expected was determined so as to allow evaluation of the worth of the experiment. Phase shift analysis was performed for Gaussian, Exponential, Yukawa, and Square Well spherically symmetric potentials. The energy range considered was 20 kev to 2.6 MeV. Scattering length was found to be determined to within 6% and effective range within 40% for a 10% scattering cross section accuracy. Potential depth would be determined to at least .5% and range to .3%.

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1. Introduction

A low energy neutron-neutron scattering experiment utilizing a single underground nuclear explosion has been proposed and studied. [2,3] Preliminary studies indicate the scattering cross section might be measured to an accuracy of 10% from 20 kev to 2MeV. The efforts of this paper are directed to a numerical investigation of the expected scattering with special attention given to: (a) the possible existence of the dineutron; (b) the effect of changes in effective range and scattering length upon the range and depth of the spherically symmetric Gaussian, Exponential, Yukawa, and Square Well Potentials; (c) the effect of changes in effective range and scattering length upon the scattering cross section from each of the potential shapes in (b); and (d) a comparison of calculated cross sections from the four potential shapes in (b) with the shape-independent cross section.

Precise determination of the neutron-neutron scattering parameters would be important for a number of reasons as stated by Moravcsik. [3] These reasons include: (1) an investigation of charge independence; (2) an investigation of charge symmetry; and (3) the existence or non-existence of a slightly unbound dineutron resonance state.

This investigation attempted to locate a dineutron state in section 2 to provide a theoretical justification for its experimental quest. In Sections 3 and 4, 2 to 16% deviations in the ratio of effective range to scattering length from the accepted n-p ratio were investigated for their effect on scattering cross sections and Gaussian, Exponential, Yukawa, and Square Well Potential parameters. These results should allow some determination of the accuracy in experimental results required to determine to a given accuracy the shape-independent scattering parameters.

In Section 5, the cross section from effective-range theory is compared with cross sections from potential scattering to test the validity of the shape independent approximation. This report concludes with a summary in section 6 and the complete computer program used in the calculations along with the calculation results as a part of the appendix.

The analysis was performed using phase shift scattering methods. The well established scattering length and effective range for neutron-proton scattering, a^*_s and $r_{o,s}^*$, were taken as a base for calculations. The ratio of effective range to scattering length was formed to yield a spherically symmetric potential range and depth for square well, exponential, gaussian, and yukawa potentials. The absolute value of the scattering length was then varied in 2% steps up to 16% while holding the effective range constant at its accepted value. 17 sets of potential parameters were thus obtained for each of the four potential shapes. Then, using the maximum and minimum acceptable value of effective range as a base, the effective range was varied in 2% increments up to 6% while holding the scattering length constant at its accepted value. Eight additional sets of potential parameters were obtained for each of the potential shapes.

The spherically symmetric radial wave equation was then numerically integrated using a fourth order runge kutta method for all calculated sets of parameters and potential shapes. Integration was performed for 30 values of center of mass energy ranging from 20 kev to 2.6MeV and was carried out to a radius at which the effect of the nuclear potential was no longer felt.

The logrithmic derivative of the numerically integrated solution was then compared to the logrithmic derivative of the exact field-free asymptotic solution outside the range of the nuclear potential and a

phase shift obtained.

The total cross section as a function of center of mass energy was then calculated from the phase shift for each case. A ratio of each cross section to that obtained from the accepted neutron-proton scattering potential parameters was formed. This gave the per cent change in total cross section for each incremental change in scattering length and effective range.

Per cent change in total cross section from that for n-p scattering parameter calculations was plotted against energy for significant increments in two different energy ranges, 20 kev to 100 kev and 100 kev to 2.6MeV. Potential depth and range were plotted against the ratio of effective range to scattering length for each potential shape. A comparison of shape-independent effective range scattering calculations with potential scattering calculations was made by plotting the total cross section for each case on the same graph.

2. Possible dineutron state

An attempt was made to justify the possible existence of a dineutron resonance state near zero energy using a compound nucleus model, but failure was met in attempting to formulate a compound nucleus model for neutron-neutron scattering.

Numerical calculations for neutron-neutron potential scattering were made for a large number of potential depths on the Control Data Corporation 1604 digital computer in an attempt to find possible resonances, particularly a sharp resonance. Gaussian, exponential, and yukawa potentials were used with potential depths ranging from 0 to the value obtained from the accepted n-p scattering parameters in 2.5 MeV steps over two center of mass energy ranges, 20 to 100 kev and .1 to 2.6 MeV. Results were scanned for resonances. No resonances were expected and none were found.

3. Effective range and potential parameters

Detailed comparisons with experiments have given for the singlet effective range and scattering length for neutron-proton scattering:

$$r_{o,s}^{*} = 2.46 \pm 0.12 \text{ FM} \qquad a_s^{*} = -23.677 \pm 0.029 \text{ FM} \quad [4]$$

For an examination of the effect of deviations from this scattering length on the scattering cross section, the ratio $r_{o,s} / a_s$ was formed and a_s was varied in magnitude from 0-16% in 2% steps while holding $r_{o,s}$ at its accepted value, 2.46 Fm. To examine the effect of deviations in effective range on the scattering cross section, the ratio $r_{o,s} / a_s$ was again formed using the accepted value for a_s, -23.677 Fm, and the maximum and minimum values for $r_{o,s}$,2.58 Fm and 2.34 Fm respectively. $r_{o,s}$ was varied up to 6% in 2% steps in the direction of increasing deviation from the accepted value, 2.46 Fm.

Four spherically symmetric potential shapes were then examined. These potentials are:

Square Well:

$$V = -V_o \qquad r \leq r_n$$

$$V = 0 \qquad r > r_n$$

Gaussian Well:

$$V = -V_o \ \exp\left[-(r/r_n)^2\right]$$

Yukawa Well:

$$V = -V_o \ \frac{\exp[-r/r_n]}{r/r_n}$$

Exponential Well:

$$V = -V_o \ \exp[-r/r_n]$$

Blatt and Jackson have arrived at a correspondence between the ratio of effective range to scattering length and the potential parameters, V_0 and r_n, for each of the well shapes investigated. [1] The well parameters are given in terms of the intrinsic range, b, and the well depth parameter, S. The well depth, $V_{(r)}$, for the potential in question is multiplied by a constant such that the first resonance occurs at zero energy. The well depth parameter is then the inverse of this constant given by:

$$V(r) = s\,V_{RES}(r)$$

The intrinsic range, b, is determined by the shape and range of the adjusted potential through the radial wave function for that potential. It is defined by:

$$b \equiv r_{0_{RES}} = 2\int_0^{\infty}[1 - u_{0_{RES}}^2(r)]\,dr$$

Blatt and Jackson provide interpolation formulas to relate V_0 and r_n to r_0 and a. The form of their formulas have been changed slightly to fit the definition of the potentials in this report. In their modified form, these interpolation formulas are:

Square Well:

$$S = [1 - 1.574(r_0/a) + .353(r_0/a)^2]/[1 - 2.384(r_0/a) + 1.30(r_0/a)^2]$$

$$b = r_0/[1 - .500(s-1) + .486(s-1)^2 - .476(s-1)^3 + .378(s-1)^4 - .158(s-1)^5]$$

$$V_0 = 1.02276 \times 10^{-24}\,s/b^2$$

$$r_n = b$$

Gaussian Well:

$$s = [1 - 2.030(r_0/a) + .587(r_0/a)^2]/[1 - 2.810(r_0/a) + 1.805(r_0/a)^2]$$

$$b = r_0/[1 - .641(s-1) + .568(s-1)^2 - .721(s-1)^3 + 1.172(s-1)^4 - 1.015(s-1)^5]$$

$$V_0 = 2.29208 \times 10^{-24}\,s/b^2$$

$$r_n = [b^2/2.0604]^{1/2}$$

Exponential Well:

$$s = [1 - 2.9739(r_o/a) + 1.3602(r_o/a)^2] / [1 - 3.7134(r_o/a) + 3.1177(r_o/a)^2]$$

$$b = r_o/[1 - .904(s-1) + .745(s-1)^2 - .543(s-1)^3 + .174(s-1)^4]$$

$$V_o = 7.51541 \times 10^{-24} s/b^2$$

$$r_n = b/3.5412$$

Yukawa Well:

$$s = [1 - 3.5093(r_o/a) + 2.0354(r_o/a)^2]/[1 - 4.1348(r_o/a) + 3.7072(r_o/a)^2]$$

$$b = r_o/[1 - 1.369(s-1) + 1.093(s-1)^2 - 1.127(s-1)^3 + 1.005(s-1)^4 - .324(s-1)^5]$$

$$V_o = 3.12821166 \times 10^{-24} s/b^2$$

$$r_n = b/2.1196$$

The results of these calculations are presented both in tabular and in graphical form in tables I and II and in graphs 1 through 16. The tables are arranged to exhibit V_o and r_n as a function of the percentage deviation of r_o and a from $r^*_{o,s}$ and a^*_s, the best values from n-p scattering. The graphs depict V_o and r_n as a function of $r_{o/a}$ using the left and bottom scales respectively. Using the top and right scales, the percentage deviation of V_o and r_n from the best n-p scattering parameters is shown as a function of the percentage deviation of $r_{o/a}$ from $r^*_{o,s/a^*s}$.

Variations in potential parameters were found to be of the order of a .5% increase in $|V_o|$ and a .5% decrease in r_n for a 10% increase in scattering length. A 10% increase in effective range, however, was observed to cause a decrease in $|V_o|$ of approximately 8% while increasing r_n about 9%. This data is shown in table III.

TABLE 1

EFFECT OF CHANGES IN SCATTERING LENGTH UPON POTENTIAL PARAMETERS

r_o = 2.46 Fm

%Change in a*		SQUARE WELL		GAUSSIAN WELL		EXPONENTIAL WELL		YUKAWA WELL	
	r_o/a	$-V_o$	r_n	$-V_o$	r_n	$-V_o$	r_n	$-V_o$	r_n
-16	-.087275	16.92	2.379	38.58	1.645	130.2	.6585	56.21	1.086
-14	-.089353	16.92	2.377	38.59	1.644	130.3	.6578	56.31	1.084
-12	-.091431	16.92	2.375	38.61	1.642	130.5	.6570	56.41	1.083
-10	-.093508	16.92	2.373	38.62	1.641	130.6	.6563	56.51	1.081
- 8	-.095586	16.92	2.372	38.64	1.639	130.7	.6555	56.61	1.080
- 6	-.097664	16.92	2.370	38.66	1.638	130.9	.6548	56.71	1.078
- 4	-.099742	16.92	2.368	38.67	1.636	131.0	.6540	56.80	1.077
- 2	-.101820	16.92	2.366	38.69	1.635	131.1	.6533	56.90	1.075
0	-.103898	16.92	2.365	38.70	1.633	131.2	.6525	57.00	1.074
2	-.105976	16.92	2.363	38.72	1.632	131.4	.6518	57.09	1.072
4	-.108054	16.92	2.361	38.74	1.630	131.5	.6511	57.19	1.071
6	-.110132	16.93	2.359	38.75	1.629	131.6	.6503	57.29	1.069
8	-.112210	16.93	2.358	38.77	1.628	131.8	.6496	57.38	1.068
10	-.114288	16.93	2.356	38.78	1.626	131.9	.6489	57.48	1.066
12	-.116366	16.93	2.354	38.80	1.625	132.0	.6482	57.57	1.065
14	-.118444	16.93	2.352	38.82	1.623	132.1	.6475	57.67	1.064
16	-.120522	16.93	2.351	38.83	1.622	132.3	.6468	57.76	1.062

TABLE II

EFFECT OF CHANGES IN EFFECTIVE RANGE UPON POTENTIAL PARAMETERS

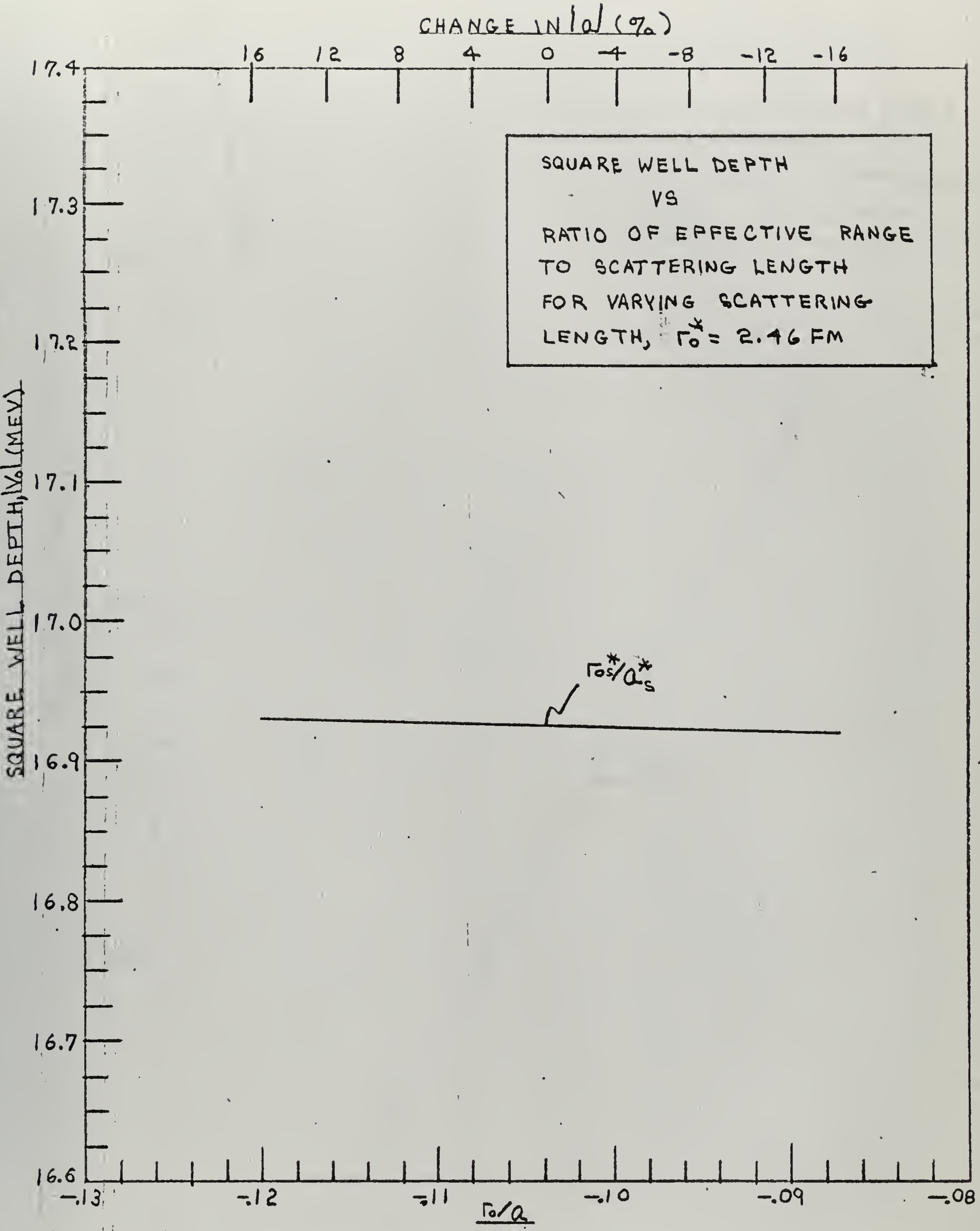
$a_s = -23.677$

%Change in r_o	r_o/a	SQUARE WELL		GAUSSIAN WELL		EXPONENTIAL WELL		YUKAWA WELL	
		$-V_o$	r_n	$-V_o$	r_n	$-V_o$	r_n	$-V_o$	r_n
60	-.166237	6.622	3.703	15.30	2.547	52.71	1.011	23.24	1.653
40	-.145458	8.644	3.263	19.91	2.247	68.24	.8939	30.03	1.464
20	-.124678	11.76	2.817	26.99	1.943	92.02	.7744	40.24	1.271
10	-.114288			32.05	1.789	109.0	.7138		
8.88	-.113325	14.23	2.571			110.8	.7081	48.28	1.164
6.88	-.111146	14.79	2.523			115.1	.6953	50.10	1.143
4.88	-.108967	15.39	2.475			119.6	.6825	52.03	1.122
0	-.103893	16.92	2.365			131.2	.6525	57.00	1.074
-4.88	-.098830	18.7	2.253			144.7	.6224	62.73	1.025
-6.88	-.096853	19.47	2.210			150.5	.6106	65.21	1.005
-8.88	-.094877	20.29	2.166			156.7	.5988	67.85	.983
-10	-.093509			47.68	1.477	161.2	.5906		
-20	-.083119	26.43	1.906	60.22	1.319	203.0	.5281	37.5	.8710
-40	-.062339	46.97	1.441	106.6	.9982	35.71	.4008	152.8	.6629
-60	.041560	105.7	.9680	238.8	.6719	794.8	.2706	337.1	.4489

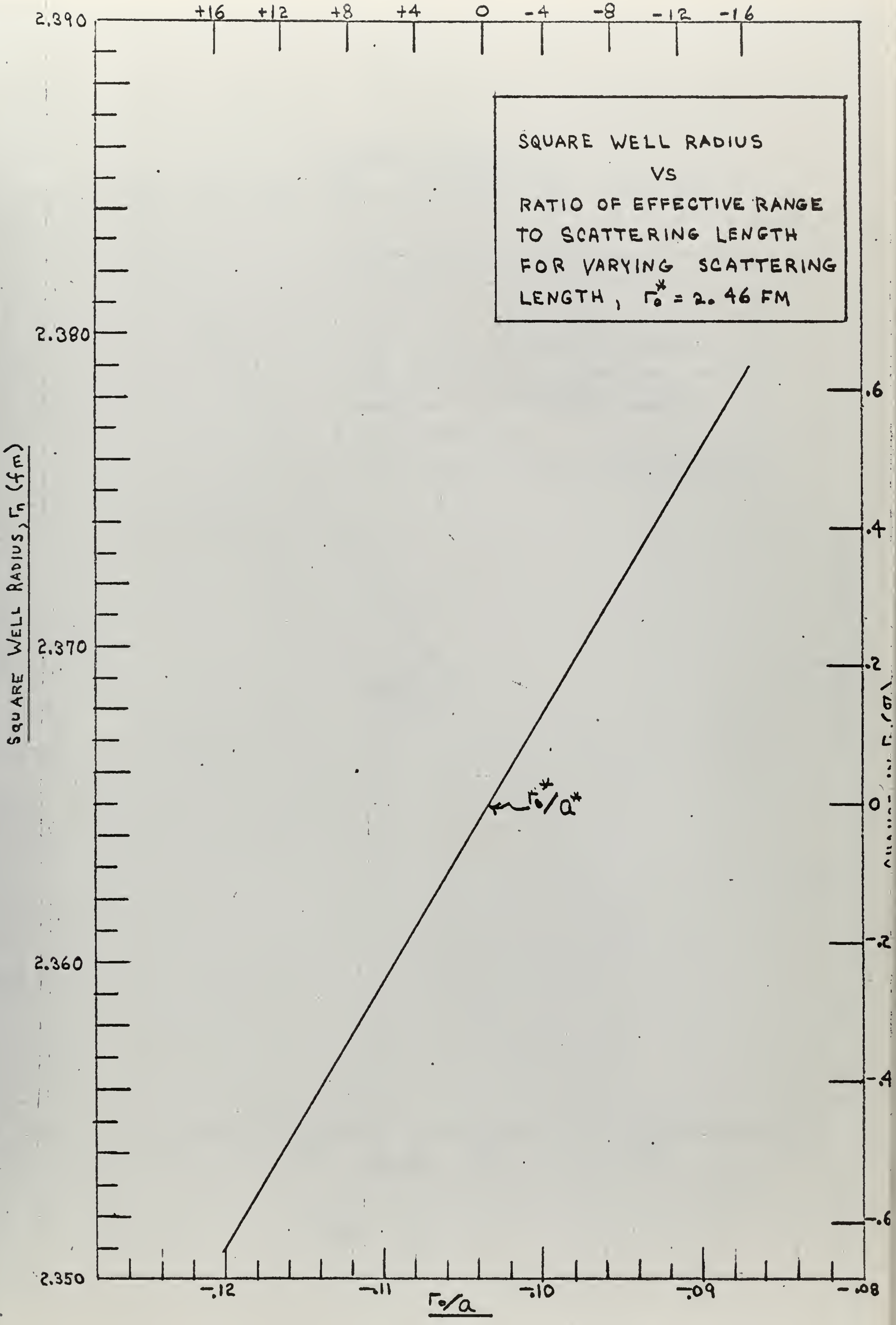
TABLE III

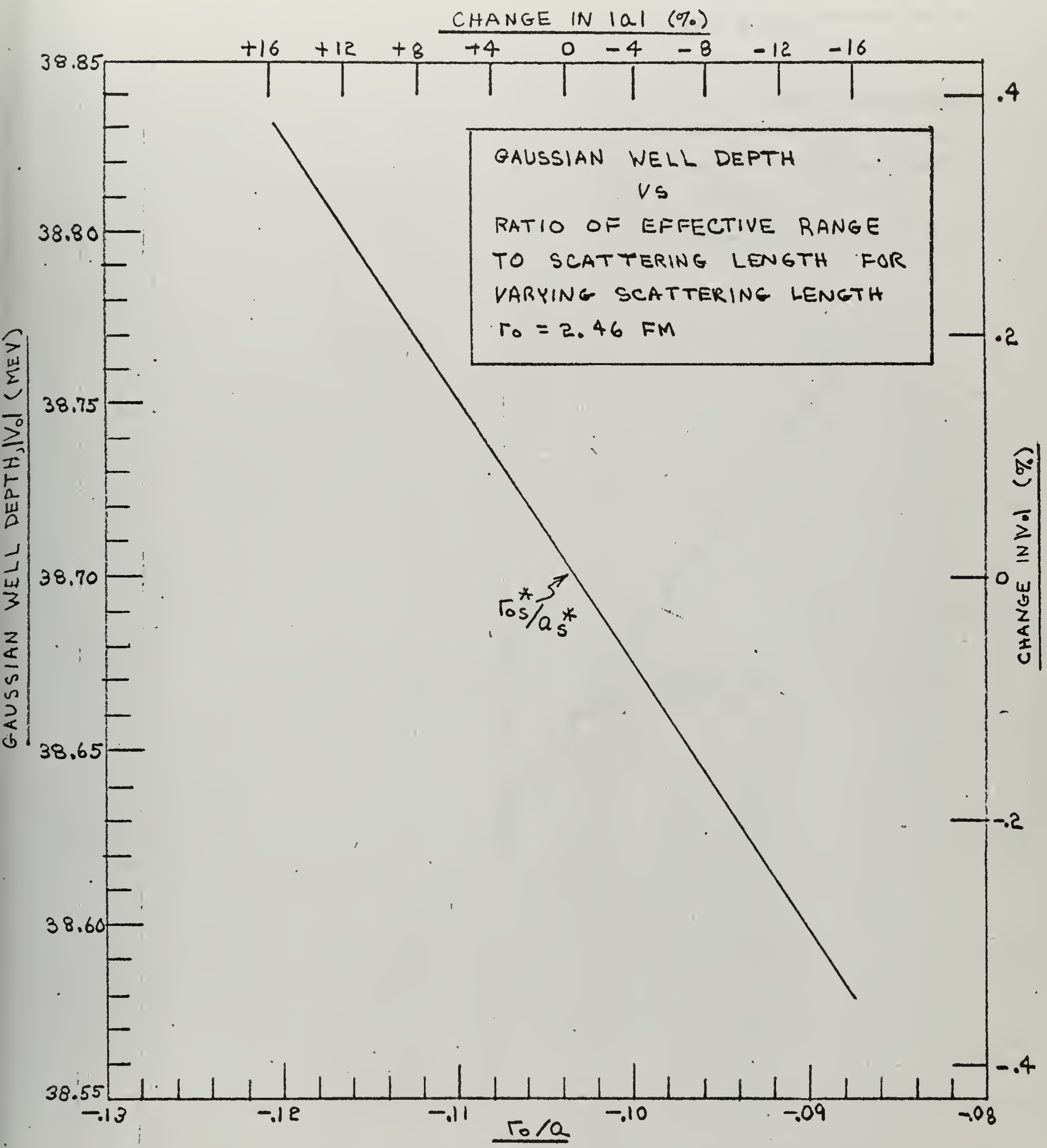
PERCENTAGE CHANGE IN WELL PARAMETERS CAUSED BY A TEN PERCENT INCREASE IN SCATTERING LENGTH AND EFFECTIVE RANGE

	$\frac{10\%\ \text{increase in } a}{\%\ \text{increase in}}$		$\frac{10\%\ \text{increase in } r_o}{\%\ \text{increase in}}$	
	$\lvert V_o \rvert$	r_n	$\lvert V_o \rvert$	r_n
SQUARE WELL	.1	-.4	-8	10
GAUSSIAN WELL	.2	-.5	-8	9
EXPONENTIAL WELL	.5	-.6	-8	9
YUKAWA WELL	.8	-.5	-8	9

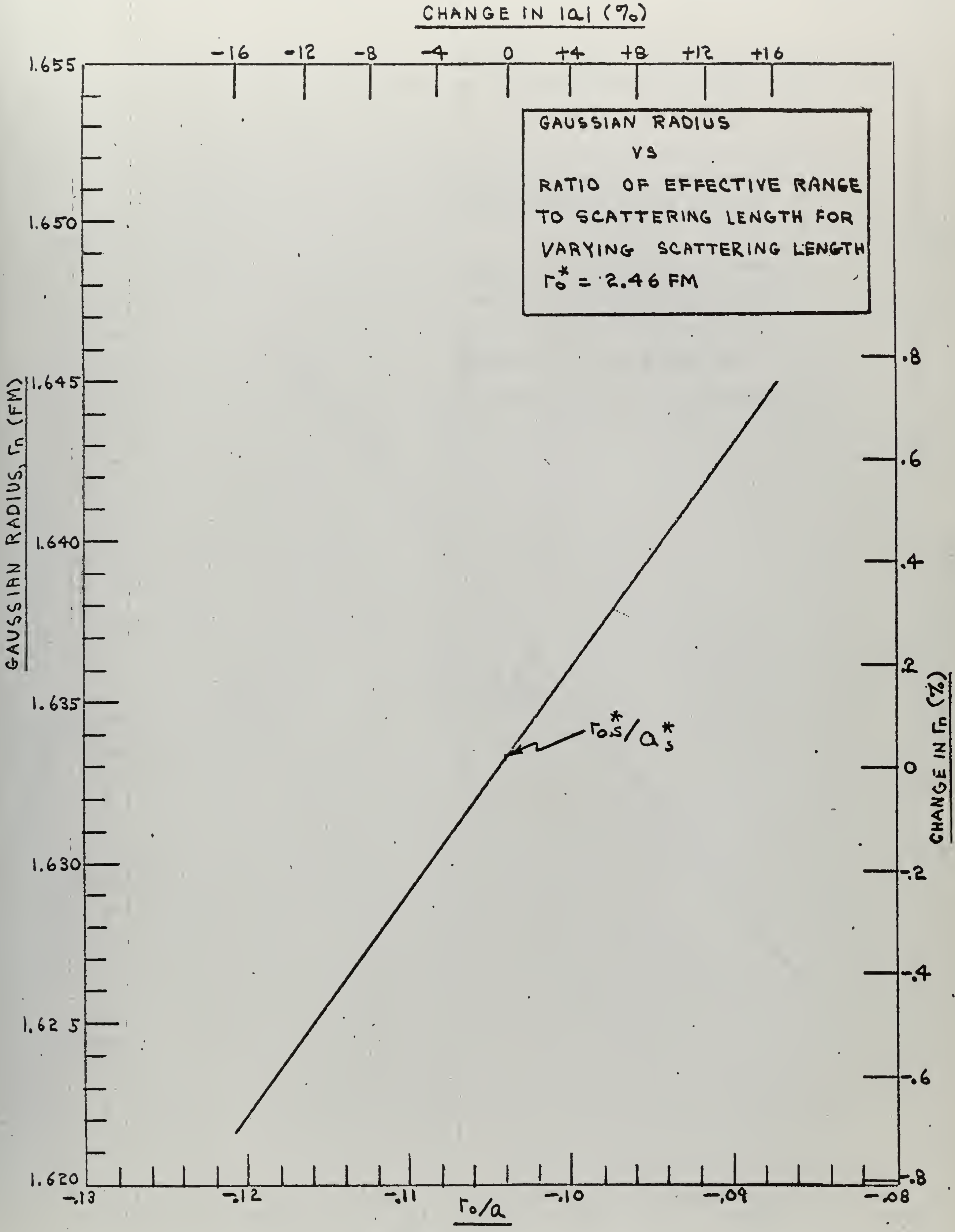

CHANGE IN |a| (%)
16
12
8
4
0
-4
-8
-12
-16
SQUARE WELL DEPTH
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH
FOR VARYING SCATTERING
LENGTH, $r_0^* = 2.46$ FM
r_{os}^*/a_s^*
SQUARE WELL DEPTH, $|V_0|$ (MEV)
17.4
17.3
17.2
17.1
17.0
16.9
16.8
16.7
16.6
-.13
-.12
-.11
-.10
-.09
-.08
r_0/a

GRAPH 1

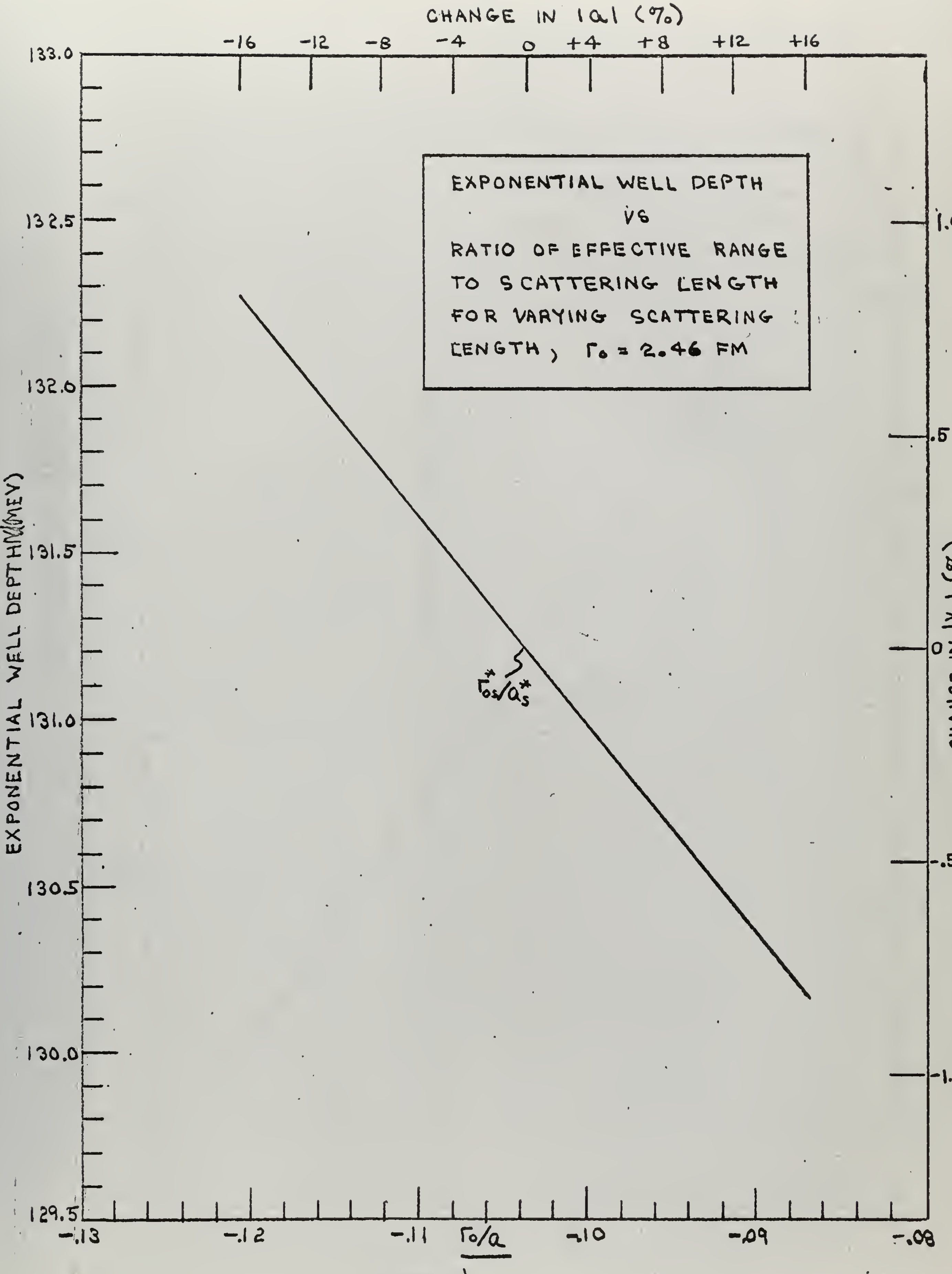

SQUARE WELL RADIUS
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH
FOR VARYING SCATTERING
LENGTH, $r_0^* = 2.46$ FM
SQUARE WELL RADIUS, r_n (fm)
2.390
2.380
2.370
2.360
2.350
+16
+12
+8
+4
0
-4
-8
-12
-16
.6
.4
.2
0
-.2
-.4
-.6
r_0^*/a^*
-.12
-.11
-.10
-.09
-.08
r_0/a


CHANGE IN |a| (%)
+16
+12
+8
+4
0
-4
-8
-12
-16
38.85
38.80
38.75
38.70
38.65
38.60
38.55
GAUSSIAN WELL DEPTH, |V₀| (MEV)
.4
.2
0
-.2
-.4
CHANGE IN |V₀| (%)
GAUSSIAN WELL DEPTH
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH FOR
VARYING SCATTERING LENGTH
r₀ = 2.46 FM
r₀s*/as*
-.13
-.12
-.11
-.10
-.09
-.08
r₀/a

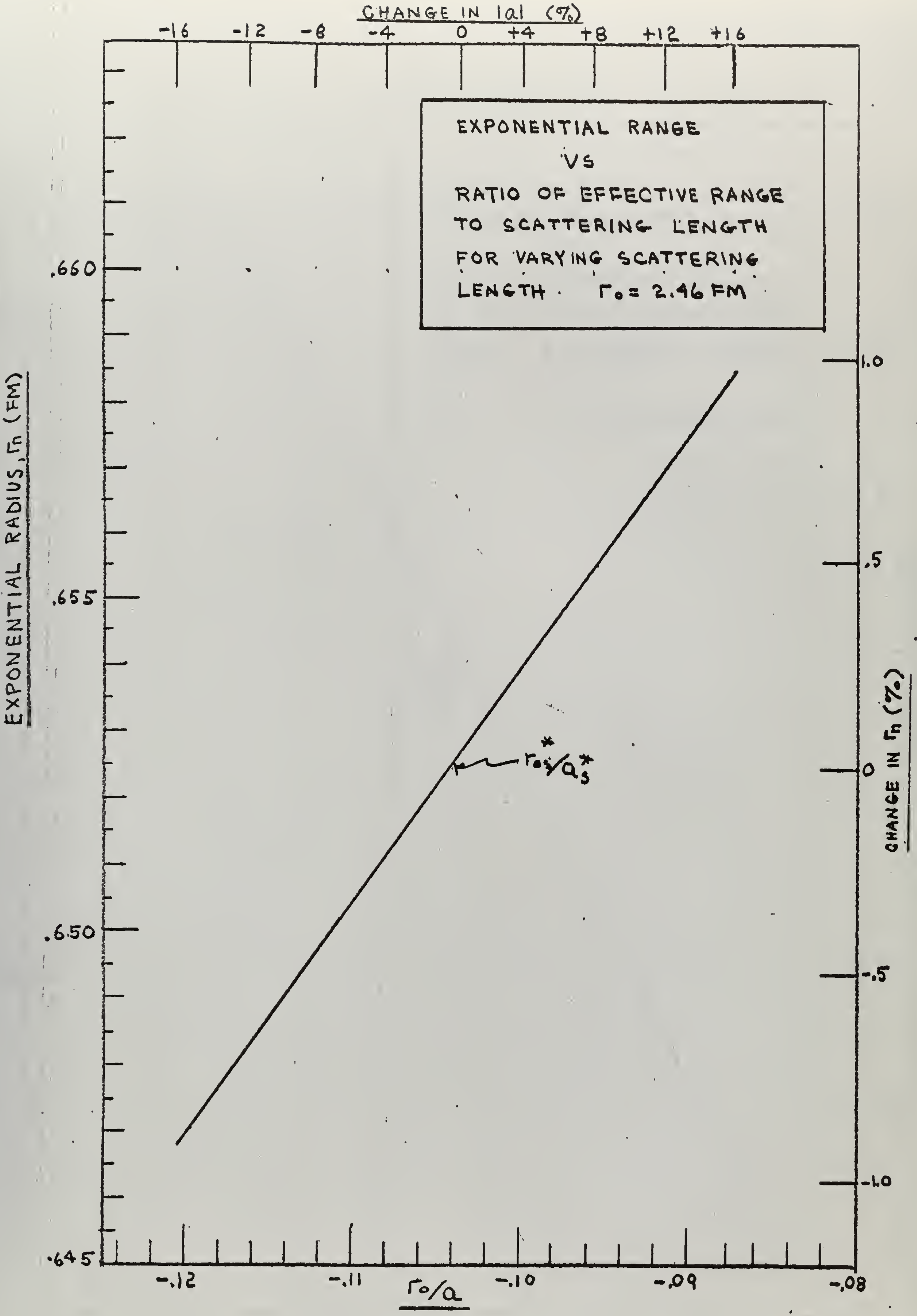
GRAPH 3


CHANGE IN |a| (%)
−16
−12
−8
−4
0
+4
+8
+12
+16
GAUSSIAN RADIUS
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH FOR
VARYING SCATTERING LENGTH
$r_o^* = 2.46$ FM
1.655
1.650
1.645
1.640
1.635
1.630
1.62 5
1.620
GAUSSIAN RADIUS, r_n (FM)
.8
.6
.4
.2
0
−.2
−.4
−.6
−.8
CHANGE IN r_n (%)
$r_{o,s}^*/a_s^*$
−.13
−.12
−.11
−.10
−.09
−.08
r_o/a

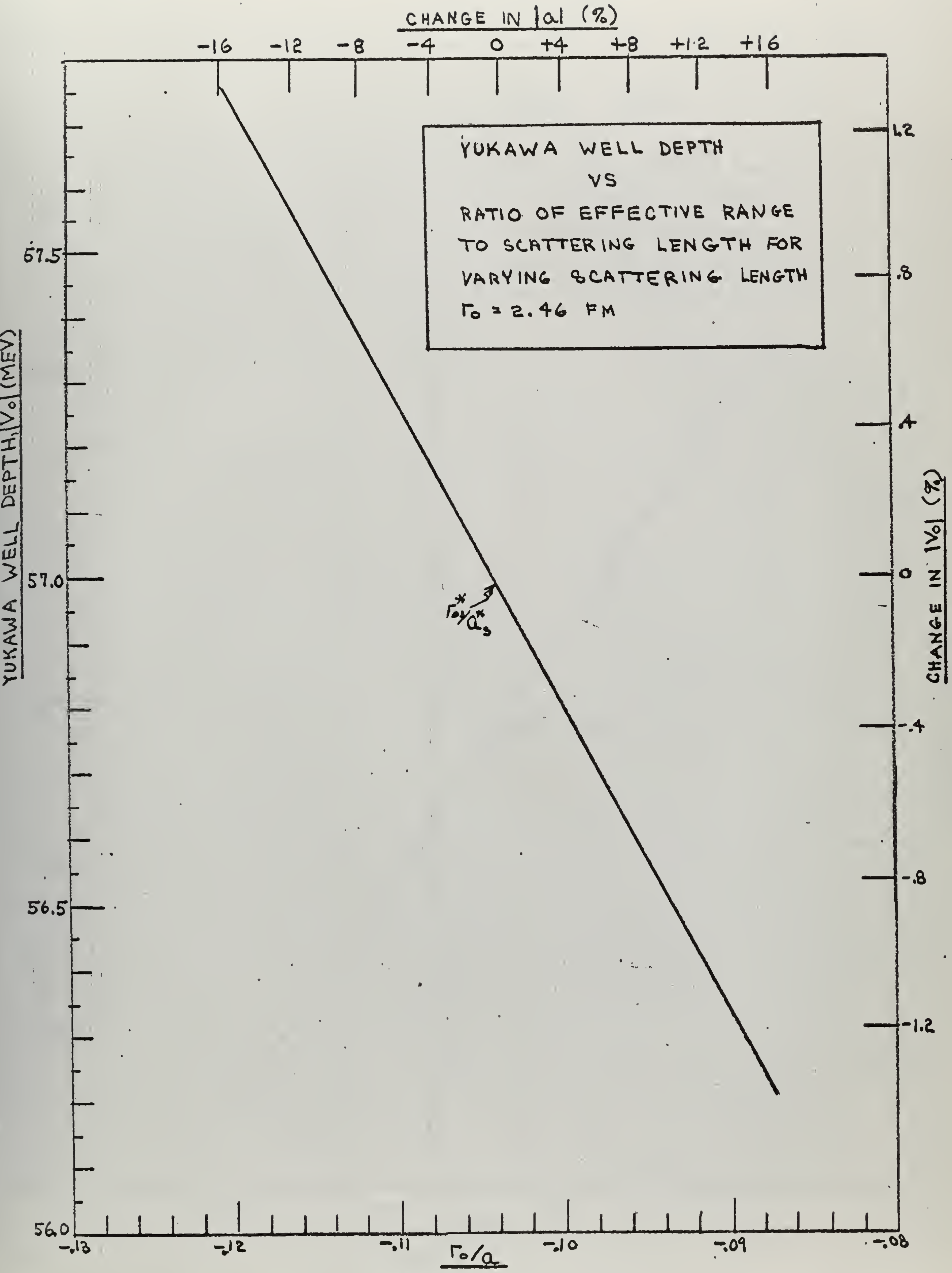
GRAPH 4


CHANGE IN |a| (%)
-16
-12
-8
-4
0
+4
+8
+12
+16
133.0
132.5
132.0
131.5
131.0
130.5
130.0
129.5
EXPONENTIAL WELL DEPTH (MEV)
EXPONENTIAL WELL DEPTH
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH
FOR VARYING SCATTERING
LENGTH, r_0 = 2.46 FM
r^*_{os}/a^*_s
1.0
.5
0
-.5
-1.0
CHANGE IN $|V_0|$ (%)
-.13
-.12
-.11
r_0/a
-.10
-.09
-.08

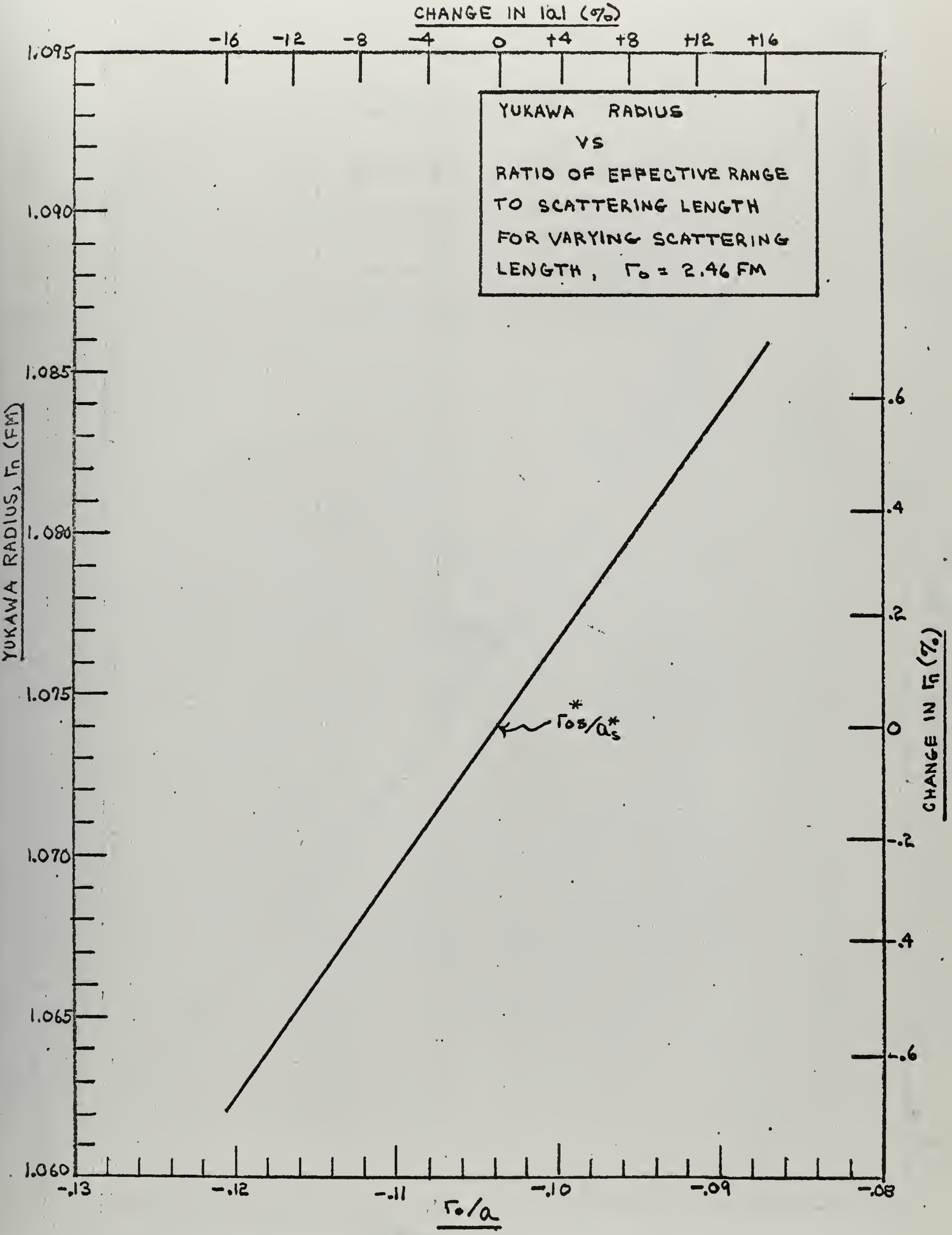
GRAPH 5


CHANGE IN |a| (%)
-16
-12
-8
-4
0
+4
+8
+12
+16
EXPONENTIAL RANGE
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH
FOR VARYING SCATTERING
LENGTH. $r_0 = 2.46$ FM
EXPONENTIAL RADIUS, r_n (FM)
.660
.655
.650
.645
1.0
.5
0
-.5
-1.0
CHANGE IN r_n (%)
r_{os}^*/a_s^*
-.12
-.11
-.10
-.09
-.08
r_o/a

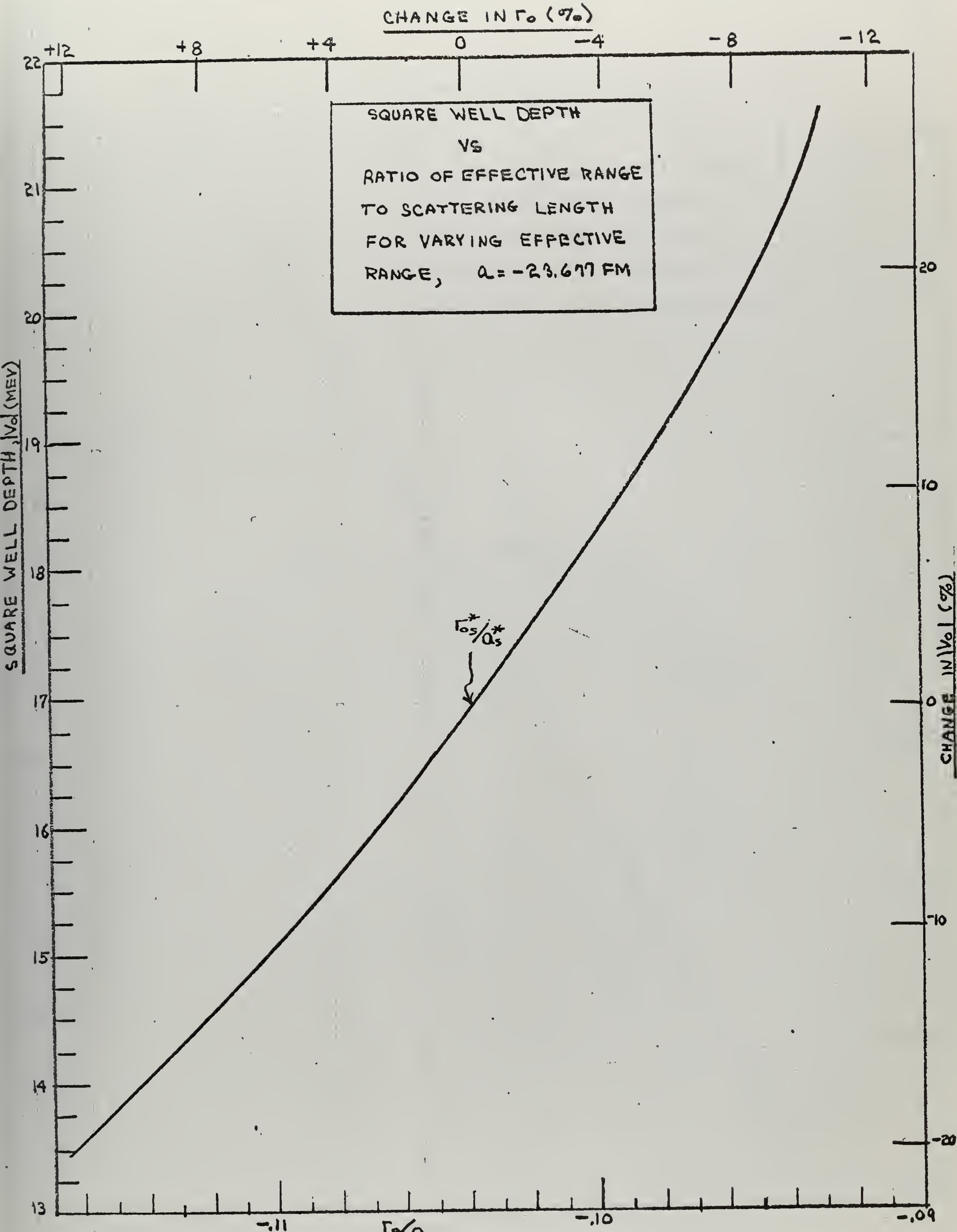
GRAPH 6


CHANGE IN |a| (%)
-16
-12
-8
-4
0
+4
+8
+12
+16
YUKAWA WELL DEPTH
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH FOR
VARYING SCATTERING LENGTH
r₀ = 2.46 FM
1.2
.8
.4
0
-.4
-.8
-1.2
CHANGE IN |V₀| (%)
YUKAWA WELL DEPTH, |V₀| (MEV)
57.5
57.0
56.5
56.0
r*₀₃/a*ₛ
-.13
-.12
-.11
-.10
-.09
-.08
r₀/a

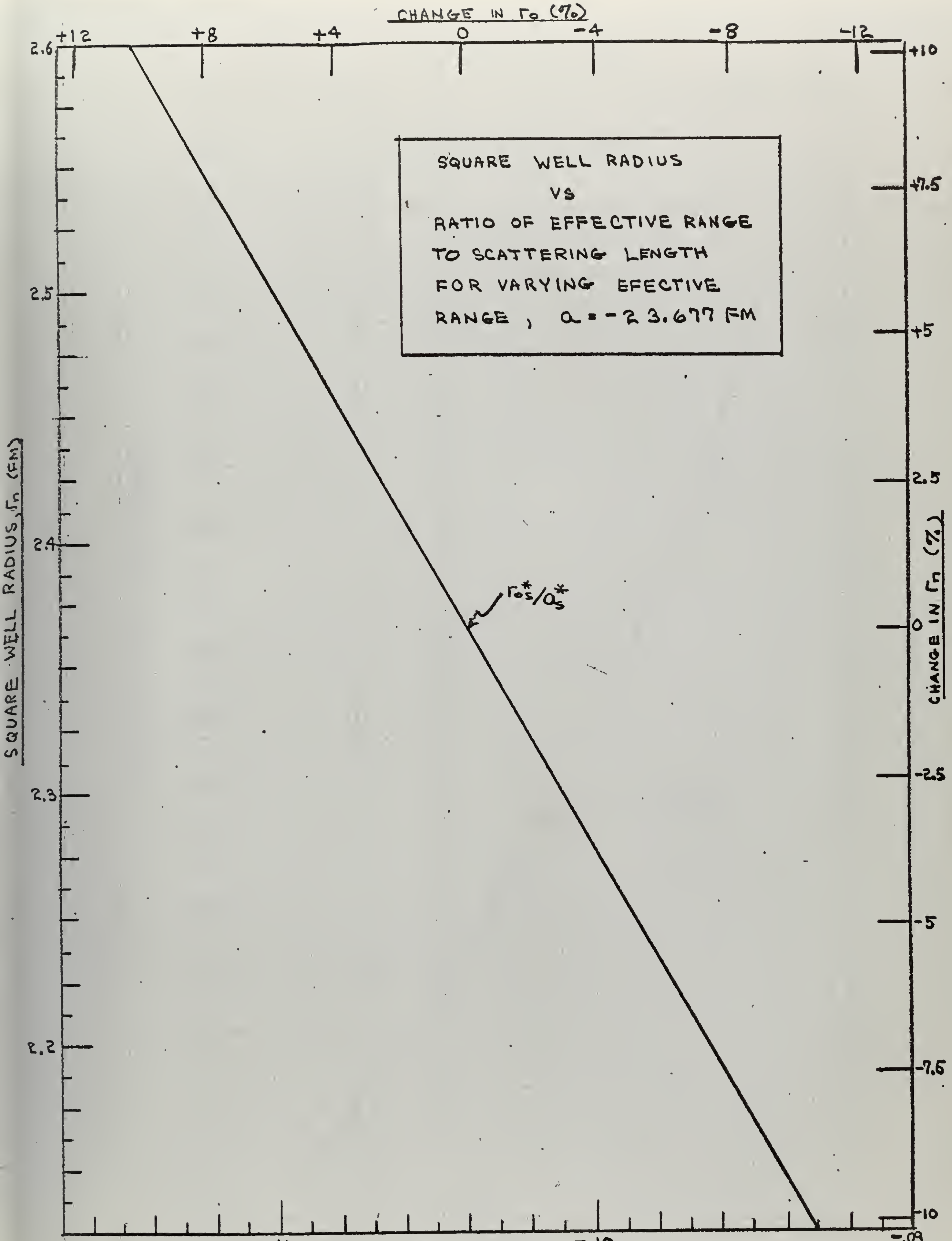
GRAPH 7


CHANGE IN |a| (%)
-16
-12
-8
-4
0
+4
+8
+12
+16
YUKAWA RADIUS VS RATIO OF EFFECTIVE RANGE TO SCATTERING LENGTH FOR VARYING SCATTERING LENGTH, r_0 = 2.46 FM
YUKAWA RADIUS, r_n (FM)
1.095
1.090
1.085
1.080
1.075
1.070
1.065
1.060
r_{0s}^*/a_s^*
CHANGE IN r_n (%)
.6
.4
.2
0
-.2
-.4
-.6
-.13
-.12
-.11
-.10
-.09
-.08
r_0/a

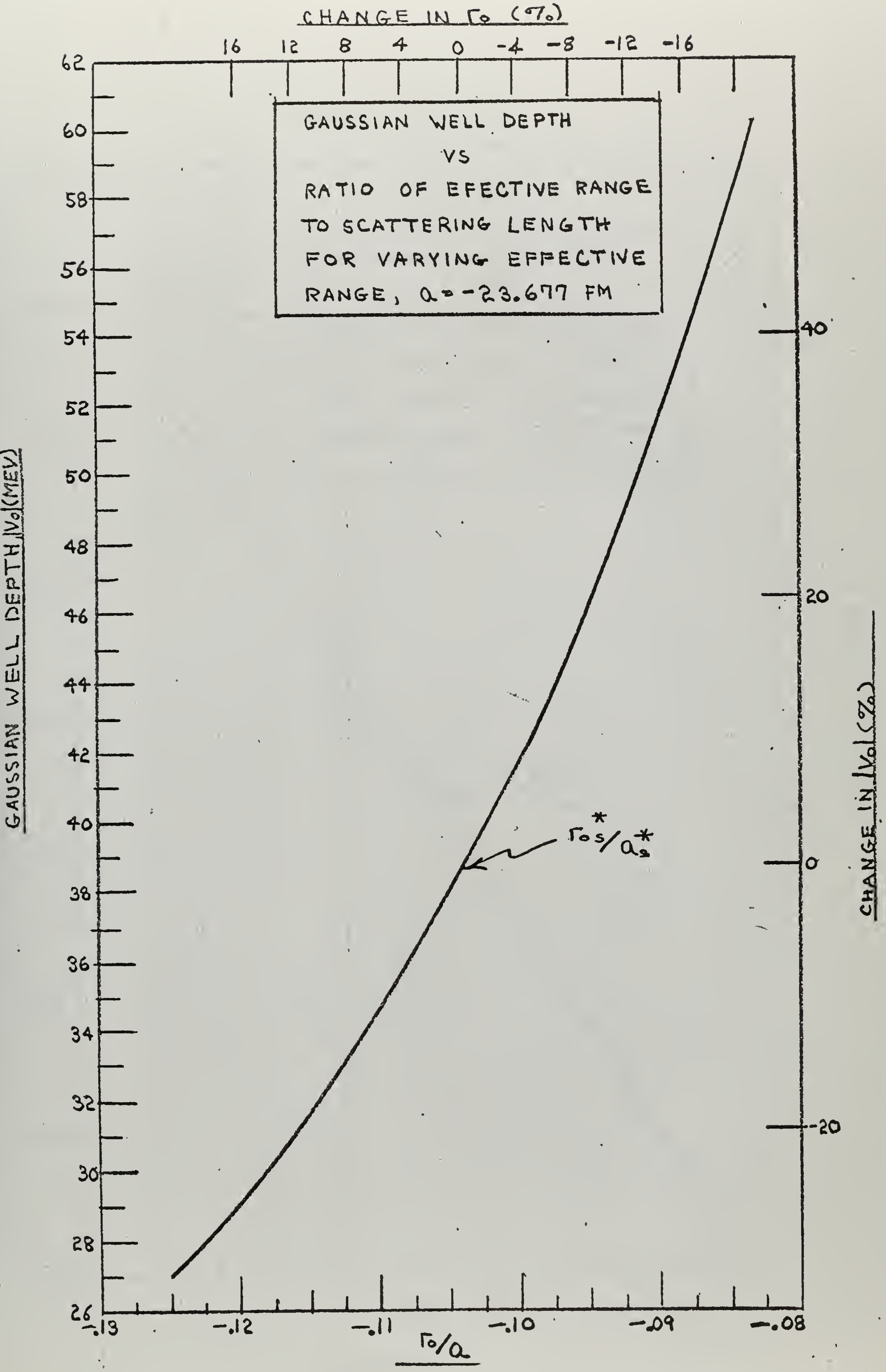
GRAPH 8


CHANGE IN r_0 (%)
+12
+8
+4
0
−4
−8
−12
SQUARE WELL DEPTH
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH
FOR VARYING EFFECTIVE
RANGE, $a = -23.677$ FM
SQUARE WELL DEPTH, $|V_0|$ (MEV)
22
21
20
19
18
17
16
15
14
13
r_{0s}^*/a_s^*
CHANGE IN $|V_0|$ (%)
20
10
0
−10
−20
−.11
−.10
−.09
r_0/a

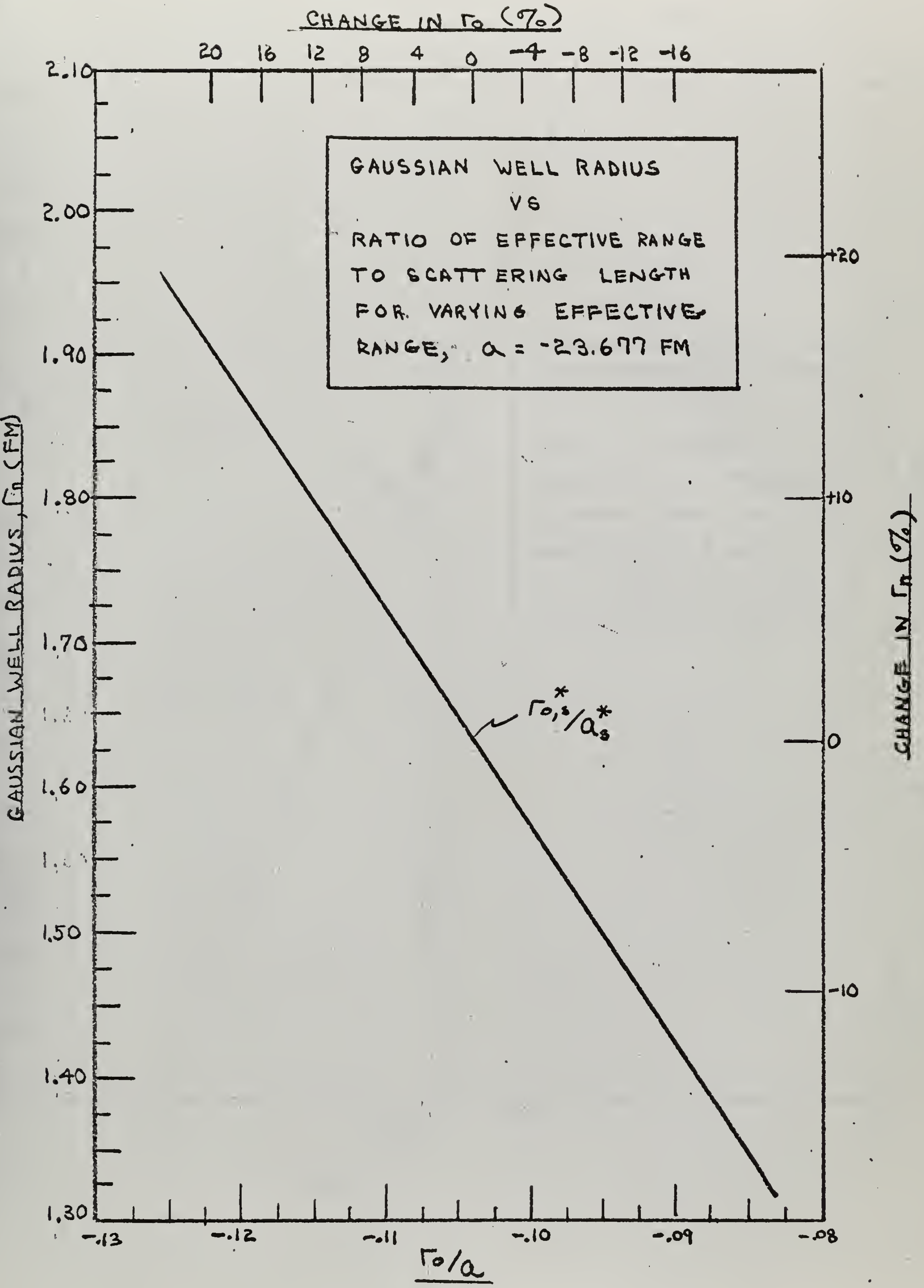
GRAPH 9


CHANGE IN r_0 (%)
+12
+8
+4
0
-4
-8
-12
2.6
2.5
2.4
2.3
2.2
SQUARE WELL RADIUS, r_n (FM)
+10
+7.5
+5
2.5
0
-2.5
-5
-7.5
-10
CHANGE IN r_n (%)
SQUARE WELL RADIUS
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH
FOR VARYING EFECTIVE
RANGE , $a = -23.677$ FM
r_{os}^*/a_s^*
-.11
-.10
-.09
r_0/a

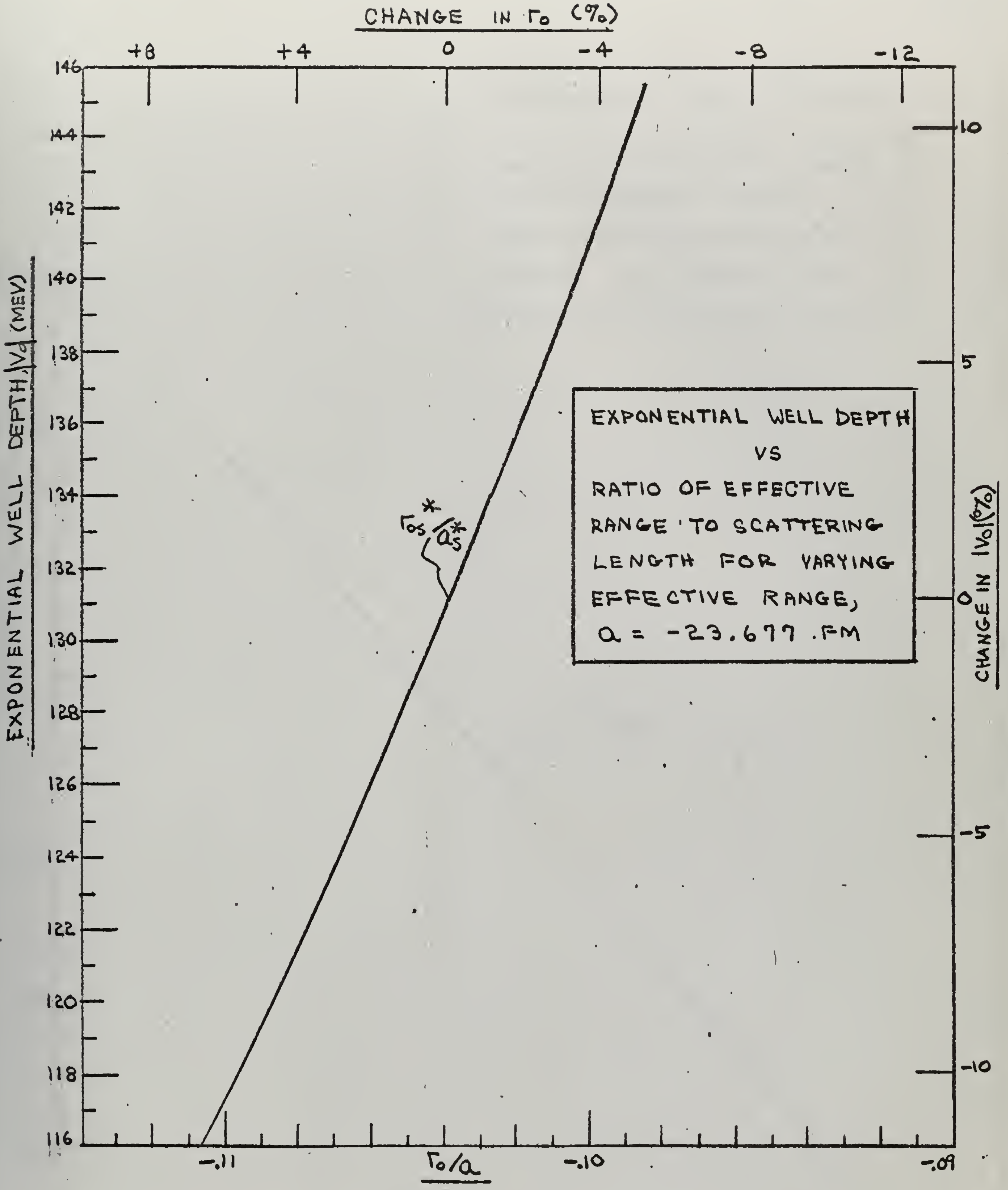
GRAPH 10


GAUSSIAN WELL DEPTH
VS
RATIO OF EFECTIVE RANGE
TO SCATTERING LENGTH
FOR VARYING EFFECTIVE
RANGE, a = -23.677 FM
CHANGE IN r_o (%)
16
12
8
4
0
-4
-8
-12
-16
GAUSSIAN WELL DEPTH $|V_o|$ (MEV)
62
60
58
56
54
52
50
48
46
44
42
40
38
36
34
32
30
28
26
CHANGE IN $|V_o|$ (%)
40
20
0
-20
r_{os}^*/a_s^*
-.13
-.12
-.11
-.10
-.09
-.08
r_o/a

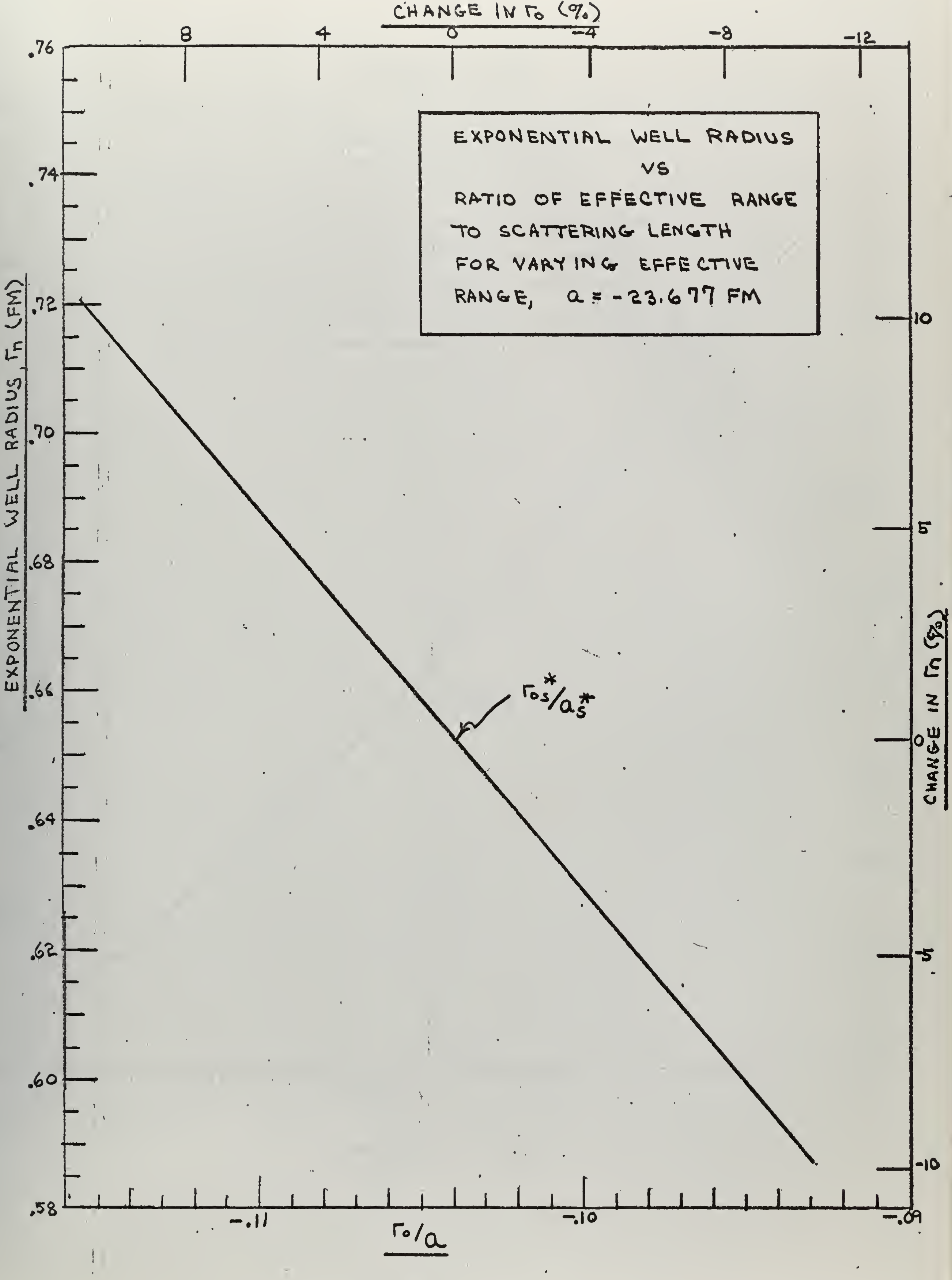
GRAPH 11


CHANGE IN r_0 (%)
20
16
12
8
4
0
-4
-8
-12
-16
GAUSSIAN WELL RADIUS VS RATIO OF EFFECTIVE RANGE TO SCATTERING LENGTH FOR VARYING EFFECTIVE RANGE, $a = -23.677$ FM
2.10
2.00
1.90
1.80
1.70
1.60
1.50
1.40
1.30
GAUSSIAN WELL RADIUS, r_0 (FM)
+20
+10
0
-10
CHANGE IN r_0 (%)
$r_{0,s}^*/a_s^*$
-.13
-.12
-.11
-.10
-.09
-.08
r_0/a

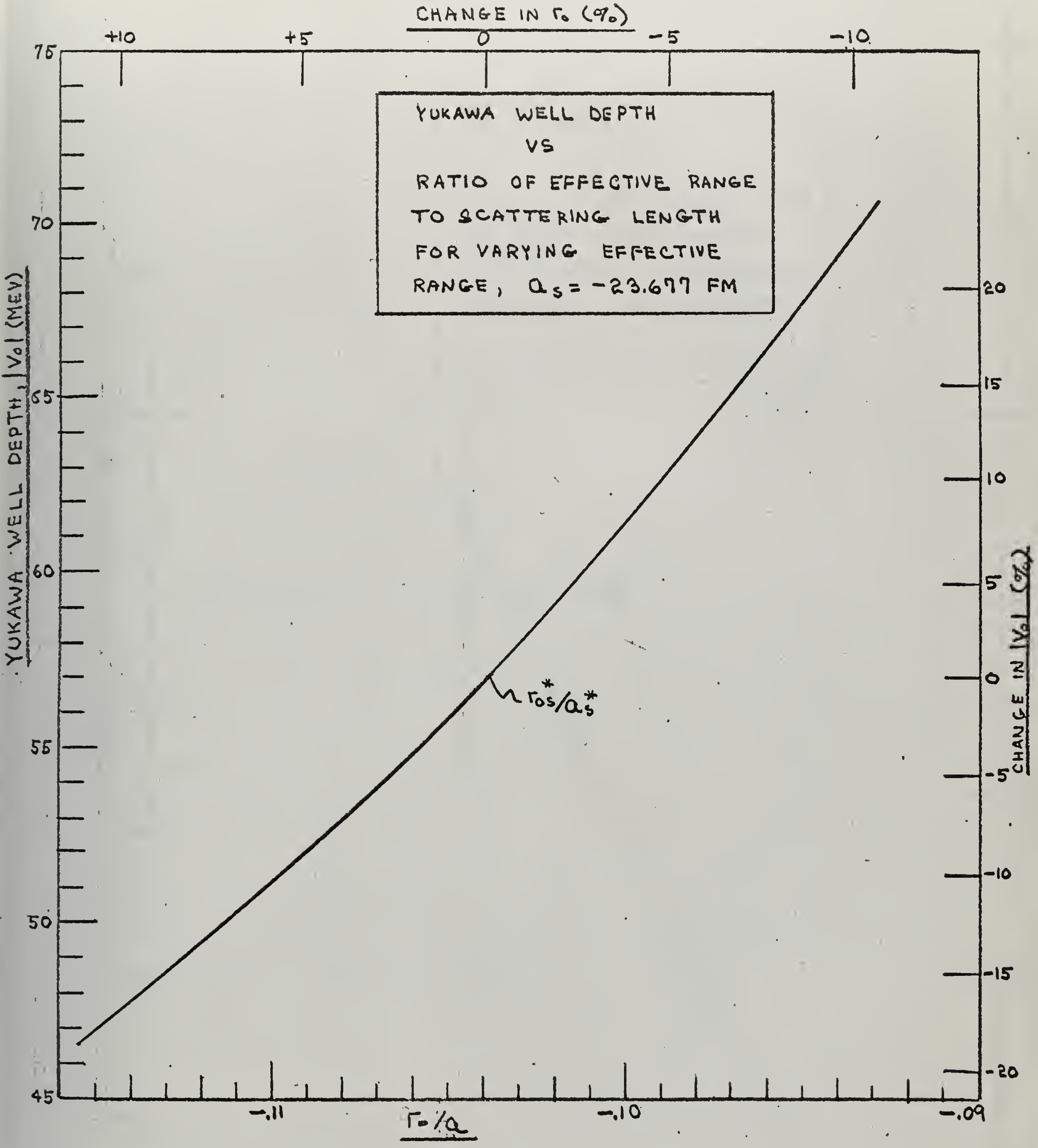
GRAPH 12


CHANGE IN r_o (%)
+8
+4
0
-4
-8
-12
146
144
142
140
138
136
134
132
130
128
126
124
122
120
118
116
EXPONENTIAL WELL DEPTH, |Vo| (MEV)
10
5
0
-5
-10
CHANGE IN |Vo| (%)
r_{os}^*/a_s^*
EXPONENTIAL WELL DEPTH
VS
RATIO OF EFFECTIVE
RANGE TO SCATTERING
LENGTH FOR VARYING
EFFECTIVE RANGE,
a = -23.677 FM
-.11
-.10
-.09
r_o/a

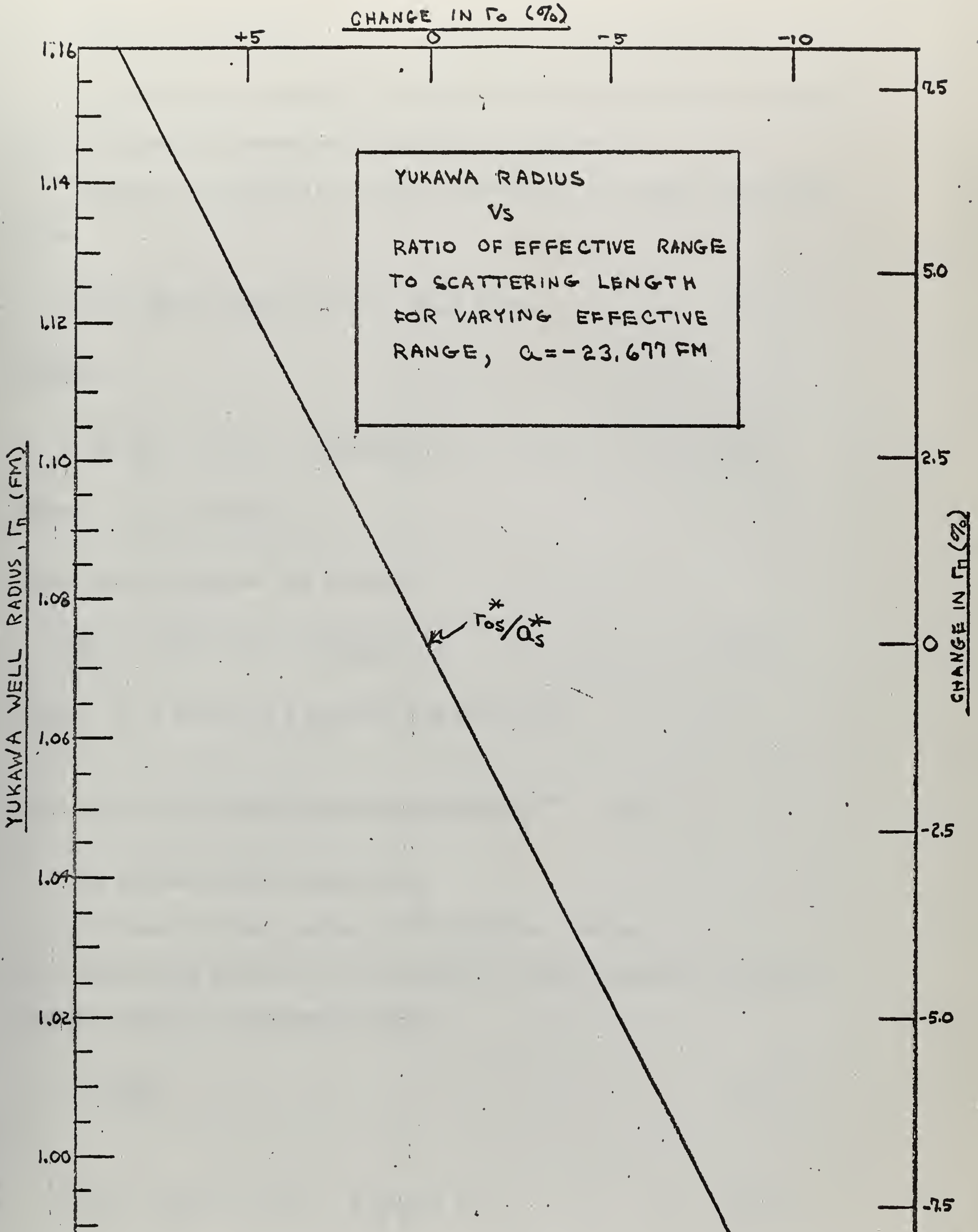
GRAPH 13


CHANGE IN r_0 (%)
8
4
0
-4
-8
-12
EXPONENTIAL WELL RADIUS
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH
FOR VARYING EFFECTIVE
RANGE, a = -23.677 FM
EXPONENTIAL WELL RADIUS, r_n (FM)
.76
.74
.72
.70
.68
.66
.64
.62
.60
.58
CHANGE IN r_n (%)
10
5
0
-5
-10
r_{os}^*/a_s^*
-.11
-.10
-.09
r_o/a

GRAPH 14


CHANGE IN r_0 (%)
+10
+5
0
−5
−10
75
70
65
60
55
50
45
YUKAWA WELL DEPTH, $|V_0|$ (MEV)
YUKAWA WELL DEPTH
VS
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH
FOR VARYING EFFECTIVE
RANGE, $a_s = -23.677$ FM
r_{os}^*/a_s^*
20
15
10
5
0
-5
-10
-15
-20
CHANGE IN $|V_0|$ (%)
−.11
−.10
−.09
r_0/a

GRAPH 15


CHANGE IN r_o (%)
+5
0
-5
-10
YUKAWA RADIUS
Vs
RATIO OF EFFECTIVE RANGE
TO SCATTERING LENGTH
FOR VARYING EFFECTIVE
RANGE, $a = -23.677$ FM
r_{os}^*/a_s^*
YUKAWA WELL RADIUS, r_n (FM)
1.16
1.14
1.12
1.10
1.08
1.06
1.04
1.02
1.00
.98
CHANGE IN r_n (%)
7.5
5.0
2.5
0
-2.5
-5.0
-7.5
-.11
-.10
-.09
r_o/a

GRAPH 16

4. Phase shift analysis

A partial wave analysis of neutron-neutron scattering was performed for all potential parameters calculated in section 3.

Assuming a spherically symmetric potential, the radial wave equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\left\{\frac{2m}{\hbar^2}[E-V(r)]-\frac{\lambda}{r^2}\right\}R=0 \qquad (1)$$

becomes

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2}+\left[V(r)+\frac{l(l+1)\hbar^2}{2mr^2}\right]u=Eu \qquad (2)$$

where $u(r)=r\,R(r)$

Rearranging, equation (2) becomes

$$\frac{d^2u}{dr^2}=-\left\{k^2-V'(r)-\frac{l(l+1)}{r^2}\right\}u \qquad (3)$$

where $k^2=ME/\hbar^2=[E(MEV)/41.499]\times10^{26}$

and $V'(r)=MV(r)/\hbar^2=[V(r)(MEV)/41.499]\times10^{26}$ [6]

M= neutron-proton reduced mass

E= center of mass energy = 1/2 laboratory energy

Separating equation (3) into two first order equations for simultaneous numerical integration gives

$$Z=\frac{du}{dr} \qquad (4)$$

$$\frac{dZ}{dr}=-\left\{k^2-V'(r)-\frac{l(l+1)}{r^2}\right\}u \qquad (5)$$

Equations (4) and (5) were numerically integrated using a fourth order Runge-Kutta method with Boundry conditions:

For $r = 0$

$$U(0) = 0$$

$$Z(0) = 1$$

Integration was carried outside the range of the nuclear potentials with 500 to 750 steps to about 15 fm for two center of mass energy ranges; 0-100 kev in 20 kev intervals, and .1-2.6 MeV in .1 MeV intervals. The values of Z and U at r = 15 fm are referred to as Z_n and U_n.

The exact asymptotic solution of the field-free wave equation, equation (1) with V' = 0, is given by Zeleny as: [7]

$$R_\ell(r) = A_\ell \left\{ \cos \delta_\ell \, j_\ell(kr) - \sin \delta_\ell \, n_\ell(kr) \right\} \tag{6}$$

from which

$$\frac{dR_\ell}{dr} = A_\ell \Big\{ \cos \delta_\ell \Big[-\frac{k}{2\ell+1} \big(\ell j_{\ell-1}(kr) - (\ell+1) j_{\ell+1}(kr) \big) \Big] - \sin \delta_\ell \Big[-\frac{k}{2\ell+1} \big(\ell n_{\ell-1}(kr) - (\ell+1) n_{\ell+1}(kr) \big) \Big] \Big\} \tag{7}$$

With the substitutions

$$U_\ell = r R_\ell$$

$$\frac{dU_\ell}{dr} = r \frac{dR_\ell}{dr} + R_\ell$$

equations (6) and (7) become:

$$U_\ell(r) = A_\ell r \left\{ \cos \delta_\ell \, j_\ell(kr) - \sin \delta_\ell \, n_\ell(kr) \right\} \tag{8}$$

$$\frac{dU_\ell(r)}{dr} = A_\ell \Big\{ r \Big[\frac{k}{2\ell+1} \cos \delta_\ell \big(\ell j_{\ell-1}(kr) - (\ell+1) j_{\ell+1}(kr) \Big] - \frac{k}{2\ell+1} \sin \delta_\ell \big(\ell n_{\ell-1}(kr) - (\ell+1) n_{\ell+1}(kr) \big) \Big\} \tag{9}$$

where δ_ℓ is the scattering phase shift and j_ℓ and n_ℓ are spherical Bessel and Neumman functions respectively.

The scattering phase shift, δ_ℓ, is obtained by matching the logrithmic deviations of the exact field-free and numerical solutions at a range where the nuclear force is negligible. This procedure yields;

For $\ell = 0$

$$\left.\frac{Z_n}{U_n}\right|_{r=15\,\mathrm{FM}} = \Big\{r\left[k\, n_1(kr)\sin\delta_0 - k\, j_1(kr)\cos\delta_0\right] + j_0(kr)\cos\delta_0$$

$$\left. - n_0(kr)\sin\delta_0\Big\} \Big/ \Big\{r\left[j_0(kr)\cos\delta_0 - n_0(kr)\sin\delta_0\right]\Big\}\right|_{r=15\,\mathrm{FM}}$$

For $\ell = 1$

$$\left.\frac{Z_n}{U_n}\right|_{r=15\,\mathrm{FM}} = \Big\{r\left[\frac{k}{3}\cos\delta_1\left(j_0(kr) - 2j_2(kr)\right) - \frac{k}{3}\sin\delta_1\left(n_0(kr) - 2n_2(kr)\right)\right]$$

$$\left. + \cos\delta_1\, j_1(kr) - \sin\delta_1\, n_1(kr)\Big\} \Big/ \Big\{r\left[\cos\delta_1\, j_1(kr) - \sin\delta_1\, n_1(kr)\right]\Big\}\right|_{r=15\,\mathrm{FM}}$$

Rearranging and taking the inverse tangent we get the following expressions for the phase shift:

$$\delta_0 = \tan^{-1}\left\{\frac{\left[\frac{Z_n}{U_n} j_0(kr) + k\, j_1(kr)\right] r - j_0(kr)}{\left[\frac{Z_n}{U_n} n_0(kr) + k\, n_1(kr)\right] r - n_0(kr)}\right\} \tag{10}$$

$$\delta_1 = \tan^{-1}\left\{\frac{\left[\frac{k}{3} j_0(kr) - \frac{2k}{3} j_2(kr) - \frac{Z_n}{U_n} j_1(kr)\right] r + j_1(kr)}{\left[\frac{k}{3} n_0(kr) - \frac{2k}{3} n_2(kr) - \frac{Z_n}{U_n} n_1(kr)\right] r + n_1(kr)}\right\} \tag{11}$$

The total cross section as a function of energy is obtained from the phase shift using the relation [8]

$$\sigma(E) = \sum_\ell \frac{4\pi}{k^2}(2\ell+1)\sin^2\delta_\ell \tag{12}$$

For identical particle scattering, equation (12) becomes, for the singlet state:

$$\sigma_s(E) = \sum_{\substack{\ell \\ \text{even}}} \frac{4\pi}{k^2} (2\ell+1) \sin^2 \delta_\ell$$

and for the triplet state:

$$\sigma_t(E) = \sum_{\substack{\ell \\ \text{odd}}} \frac{4\pi}{k^2} (2\ell+1) \sin^2 \delta_\ell$$

The computation of cross sections was made for both the singlet $\ell = 0$ state and the triplet $\ell = 1$ state. The $\ell = 1$ cross sections were observed to be at least 3 orders of magnitude smaller in the energy range under consideration and were disregarded in subsequent computations. The computer program in appendix III however, retains the capability of $\ell = 1$ calculations.

The computer program in appendix III was used in cross section calculations for all the potential parameters in section 3. First the program computed the potential parameters at the base scattering length and effective range and at percentage deviations from this base read into it in its input section. The total scattering cross section was then computed as a function of center of mass energy. The calculated cross section for each set of potential parameters was then divided by the cross section for the base parameters to normalize the cross section and present it as a deviation from the base.

The resulting data was plotted to show the percentage difference in total cross section from the calculated total cross section for n-p scattering with $a_s^* = -23.677$ fm and $r_{o,s}^* = 2.46$ fm as a function of energy for percentage deviations of a and r_o. The graphs are drawn over two distinct energy ranges, 20 - 100 kev and .1-2.6 MeV, to better portray results and separate graphs are provided for each

potential shape.

In Moravcsik's paper, the function Q(T) is defined by:

$$Q(T) \equiv \left(\frac{d\sigma}{d\Omega}\right)^{-1} = \frac{1}{a^2} + \left(1 - \frac{r_0}{a}\right)\epsilon = 4\pi/\sigma(E)$$

where $\epsilon = \frac{MT}{2}$ for T = laboratory kinetic energy and m = neutron mass. Moravcsik has determined that in a 10% experiment, a linear approximation for Q(T) would be sufficient.

The value of this straight line at $\epsilon = 0$ would give $1/_{a}2$ directly while the slope of the straight line would determine $r_{0/a}$ and the sign of a.

Moravcsik concludes that a determination of total cross section to 10% accuracy at ten suitably distributed energy points could determine the scattering length to 3% and the effective range to 50 - 70%.

Tables IV, V of this report show that a 10% determination of total cross section in the vicinity of the n-p scattering parameters will allow a 6% determination of a_s at 20 kev and a 16% determination at 400 kev while at 2.6 MeV a 16% error in scattering length has only a 2% effect on total cross section. The same tables show that a 10% determination of total cross section will allow a 40% determination in effective range at 2.6MeV while a 100% error in effective range has approximately a 2% effect at 20 kev.

An alternate method of determining the effective range parameters is therefore offered. The magnitude of the total cross section at 20 kev could be compared to supplemented data of this report to arrive at a value for a_s within 6%. Graph 17 is shown as an illustration of possible supplemental data. The magnitude of the total cross section at 2MeV or greater could, in the same manner, be used to arrive at a

TABLE IV

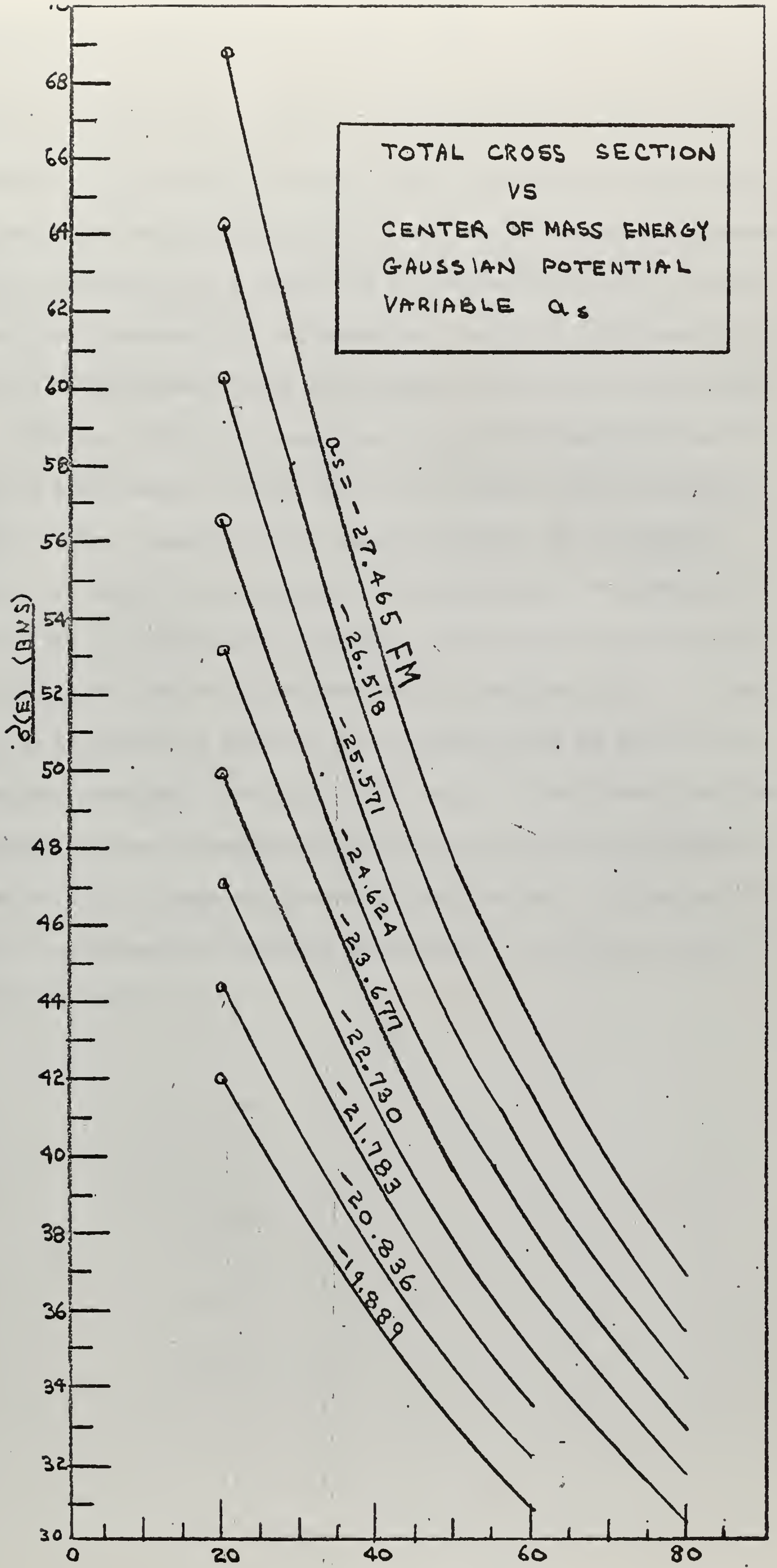
EFFECT OF A 10% ERROR IN TOTAL CROSS SECTION UPON THE ERROR IN a_s

	% Error in a_s for			
ENERGY (kev)	SQUARE WELL	GAUSSIAN	EXPONENTIAL	YUKAWA
20	8-16	6	6	6.5
60	8-16	8.5	8.5	9.5
100	8-16	11	12	16
200	8-16	16.5	16.5	18

TABLE V

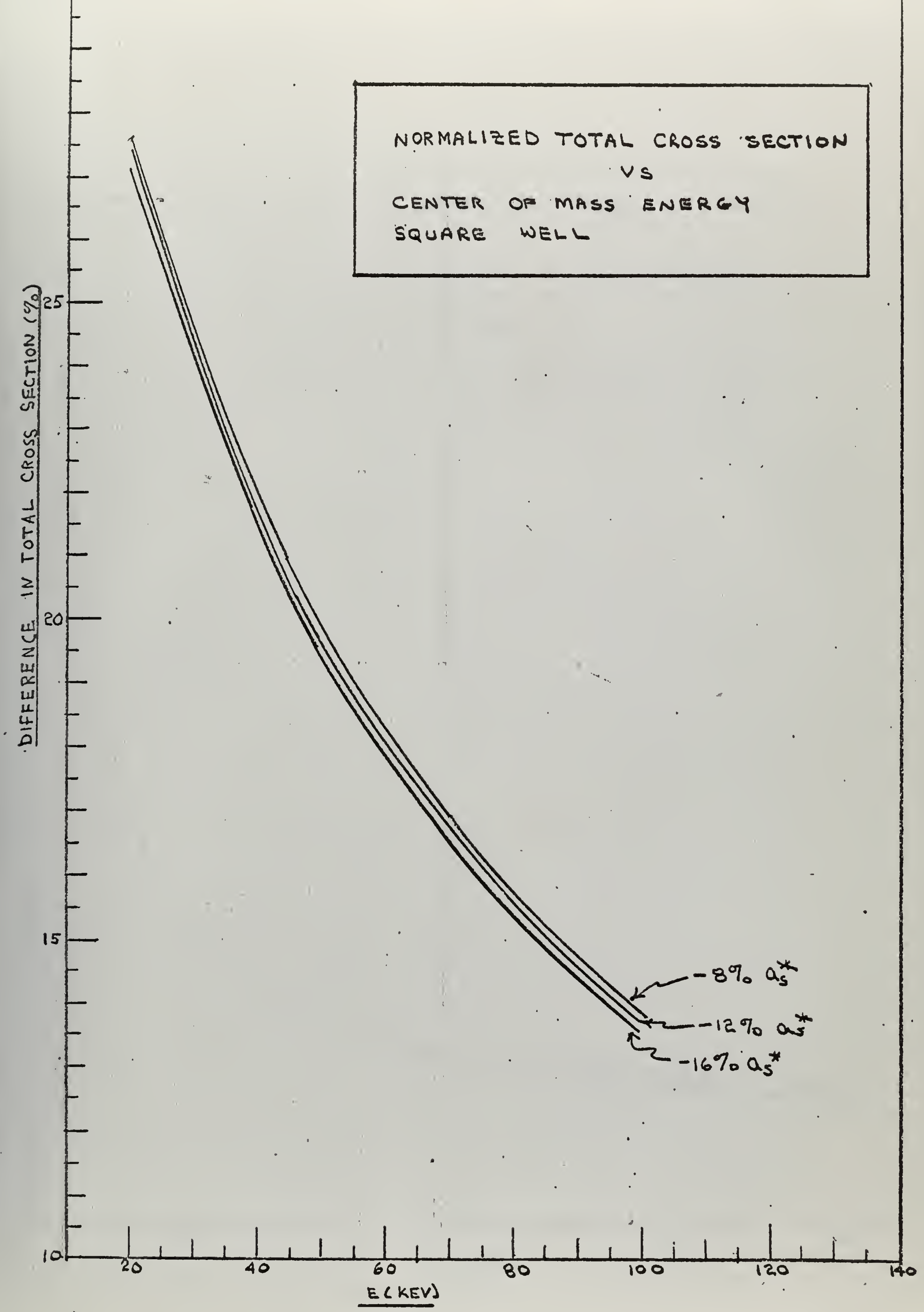
EFFECT OF A 10% ERROR IN TOTAL CROSS SECTIONS UPON THE ERROR IN r_o

	%error in $r_{o,s}$ for		
ENERGY (MeV)	GAUSSIAN	EXPONENTIAL	YUKAWA
1.6	60	58	62
1.8	56	52	59
2.0	53	50	57
2.2	50	47	55
2.4	45	45	52
2.6	40	42	50

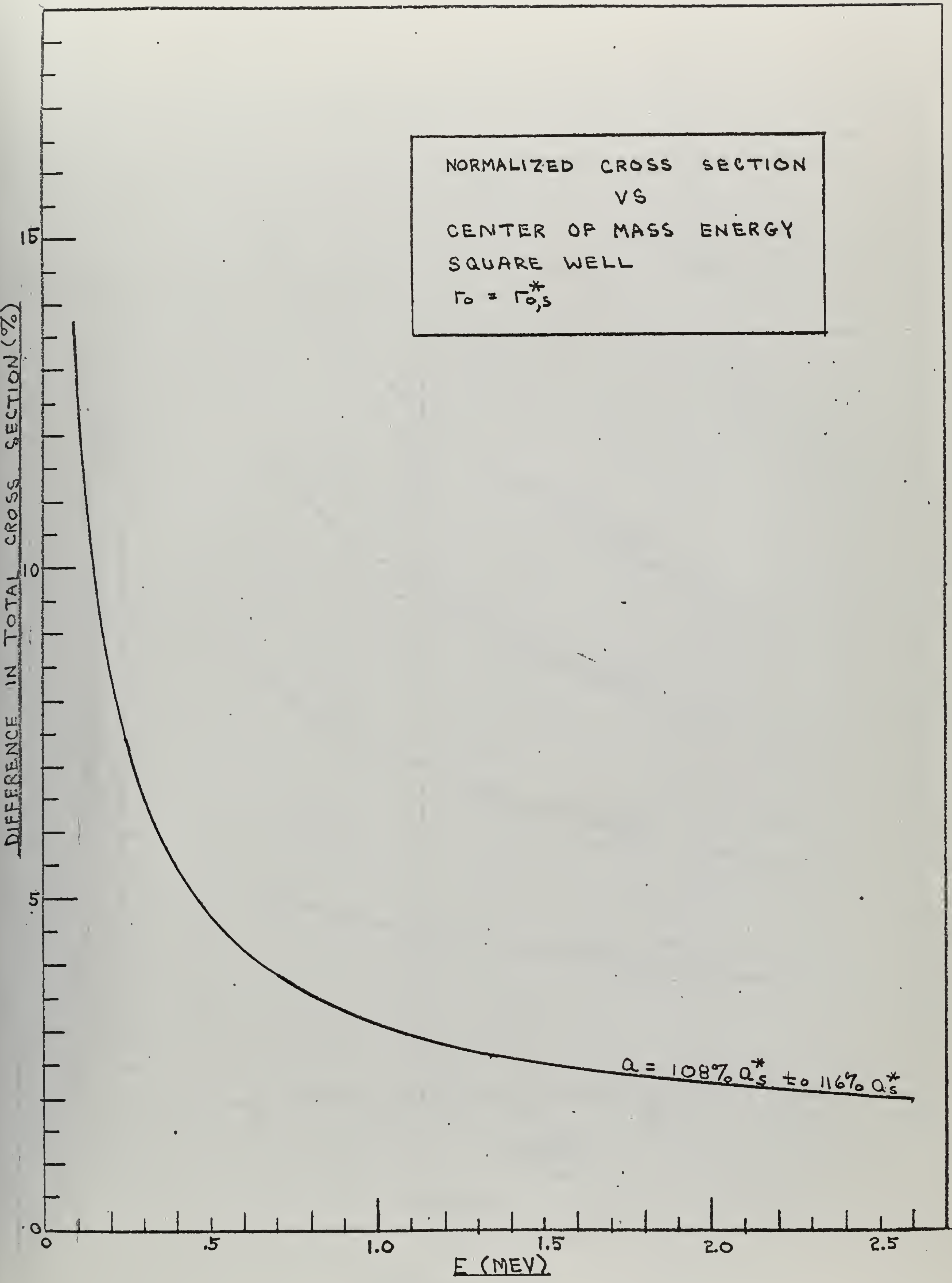
TOTAL CROSS SECTION
VS
CENTER OF MASS ENERGY
GAUSSIAN POTENTIAL
VARIABLE a_s
a_s = -27.465 FM
-26.518
-25.571
-24.624
-23.677
-22.730
-21.783
-20.836
-19.889
σ(E) (BNS)
30
32
34
36
38
40
42
44
46
48
50
52
54
56
58
60
62
64
66
68
0
20
40
60
80
E (KEV)

GRAPH 17

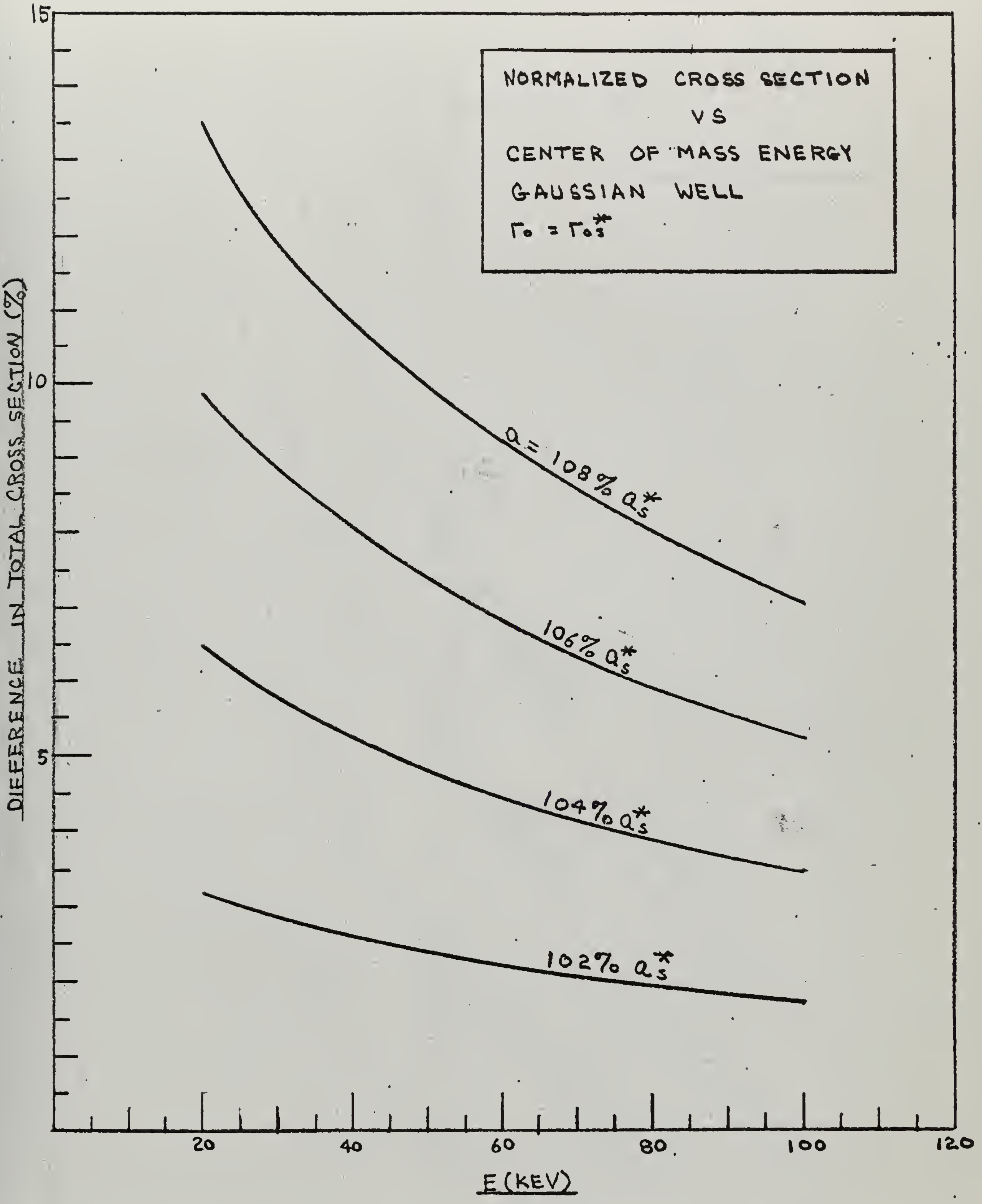
value for $r_{o,s}$ within 50%. Because of the negligible effect of both large errors in $r_{o,s}$ at the low energy cross section and large errors in a_s at the high energy cross section, they may be determined independently, each having little effect upon the determination of the other. Either gaussian, exponential, or yukawa data could be used because of only about a 1% difference in a_s value among them for any given cross section. Thus the scattering length and effective range could be determined with a minimum of two points, one at either end of the energy range. For greater accuracy, the computer program in the appendix could easily be adapted to a searching interpolation. An advantage could be gained if the necessity for only two values of cross section would allow a more precise determination of those two values. It can be seen from accompanying graphs that the energy must be accurate to 10% to achieve meaningful results in this way. If this energy accuracy is not feasible, then the method outlined by Moravcsik for scattering length and effective range determination must be used. In any event, Moravcsik's development of expected accuracies is confirmed by this phase shift analysis.


NORMALIZED TOTAL CROSS SECTION
VS
CENTER OF MASS ENERGY
SQUARE WELL
DIFFERENCE IN TOTAL CROSS SECTION (%)
25
20
15
10
20
40
60
80
100
120
140
E (KEV)
-8% a_s^*
-12% a_s^*
-16% a_s^*

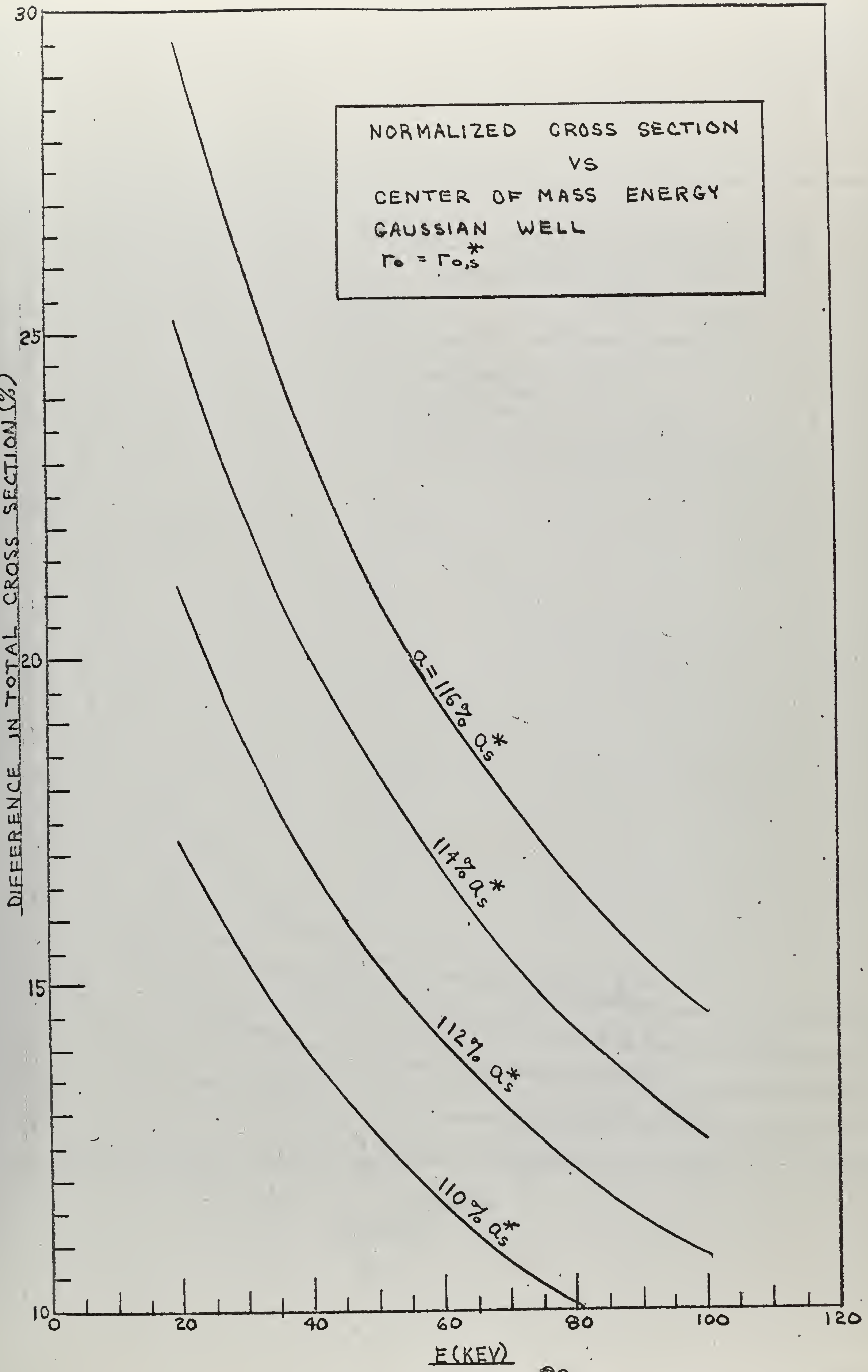
GRAPH 18


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
SQUARE WELL
$r_0 = r_{0,s}^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
15
10
5
0
0
.5
1.0
1.5
2.0
2.5
E (MEV)
$a = 108\%\, a_s^*$ to $116\%\, a_s^*$

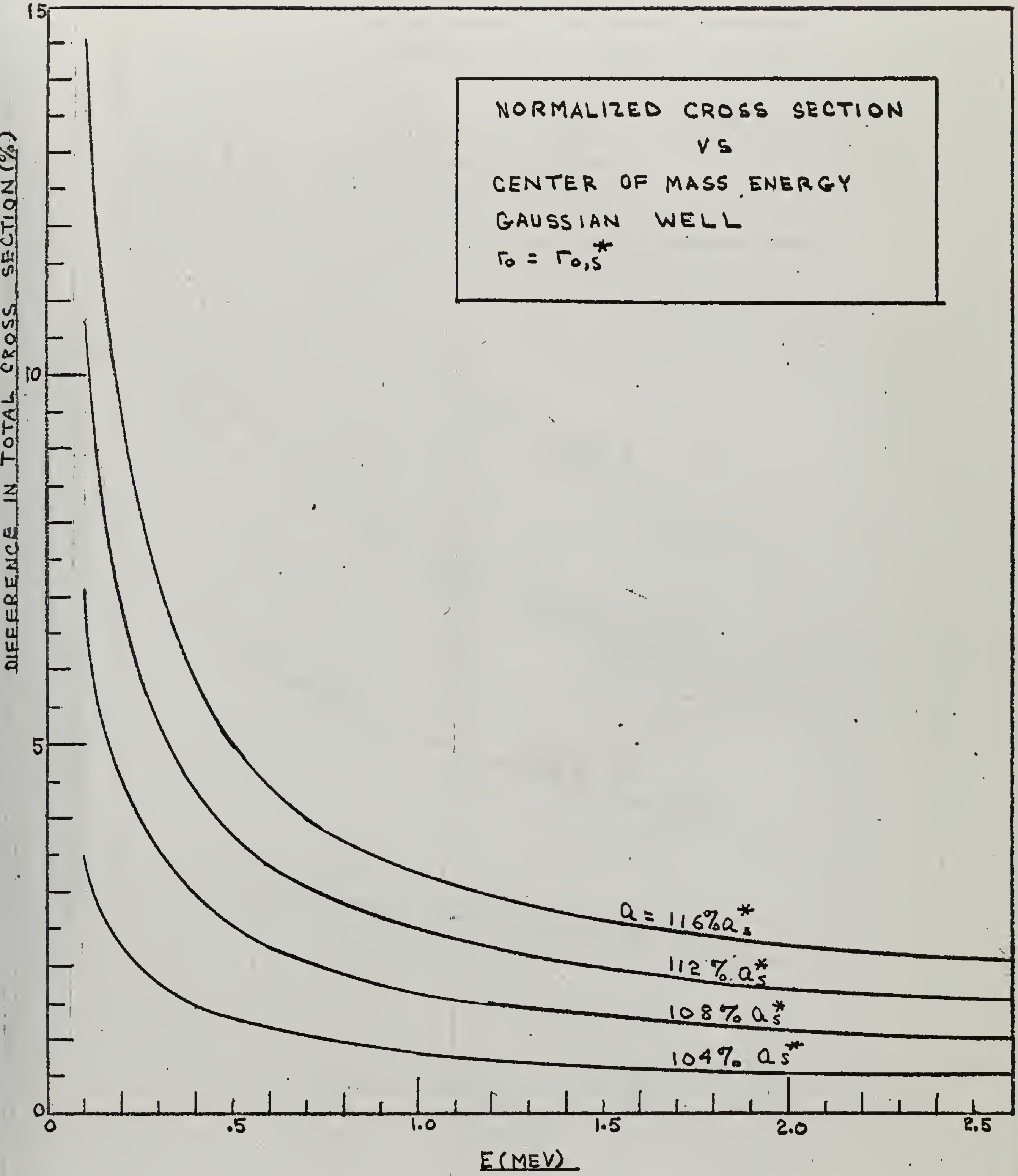
GRAPH 19


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
GAUSSIAN WELL
$\Gamma_0 = \Gamma_{0s}^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
15
10
5
$a = 108\%\ a_s^*$
$106\%\ a_s^*$
$104\%\ a_s^*$
$102\%\ a_s^*$
20
40
60
80
100
120
E (KEV)

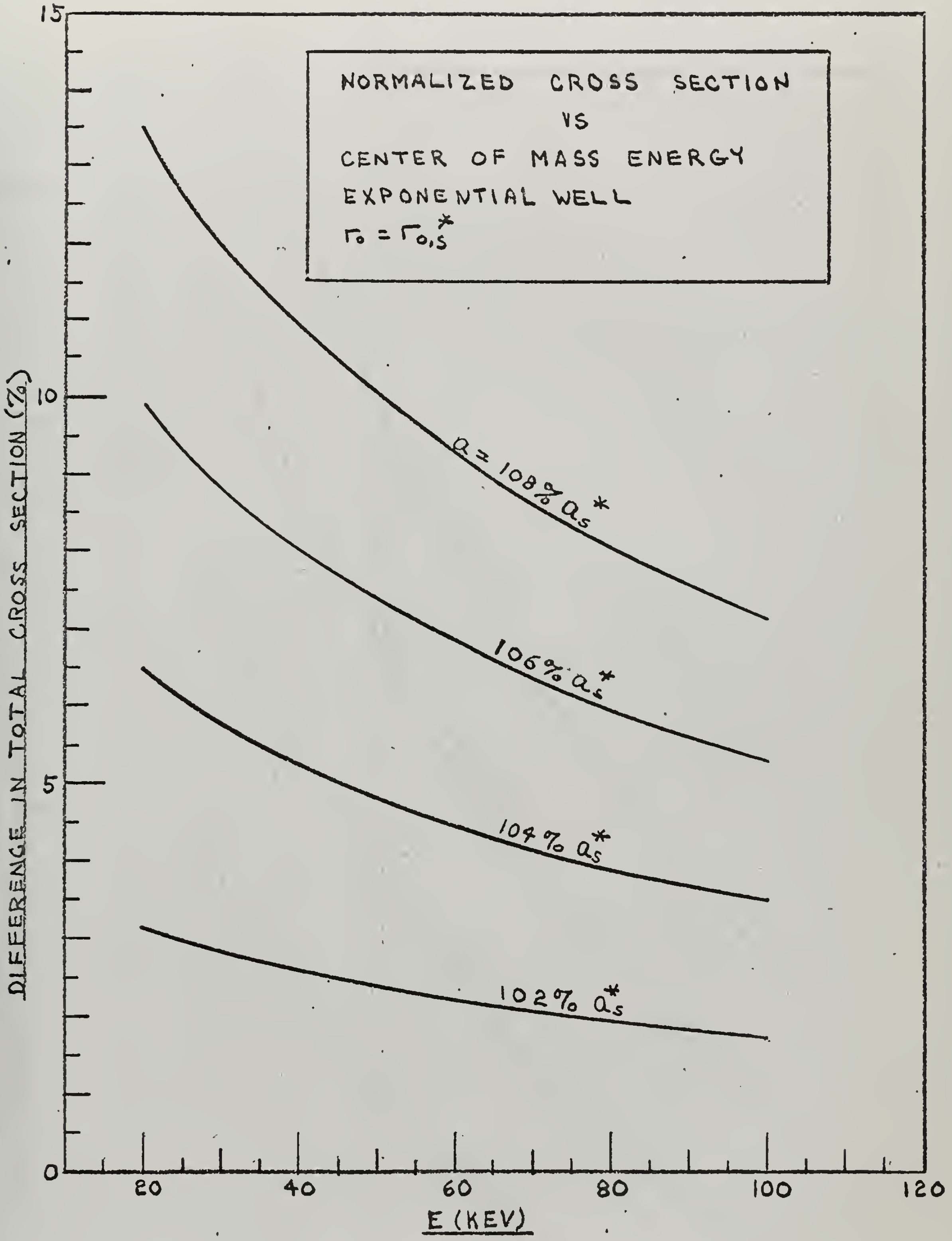
GRAPH 20


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
GAUSSIAN WELL
$r_0 = r_{0,s}^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
E(KEV)
$a = 116\% \; a_s^*$
$114\% \; a_s^*$
$112\% \; a_s^*$
$110\% \; a_s^*$
30
25
20
15
10
0
20
40
60
80
100
120

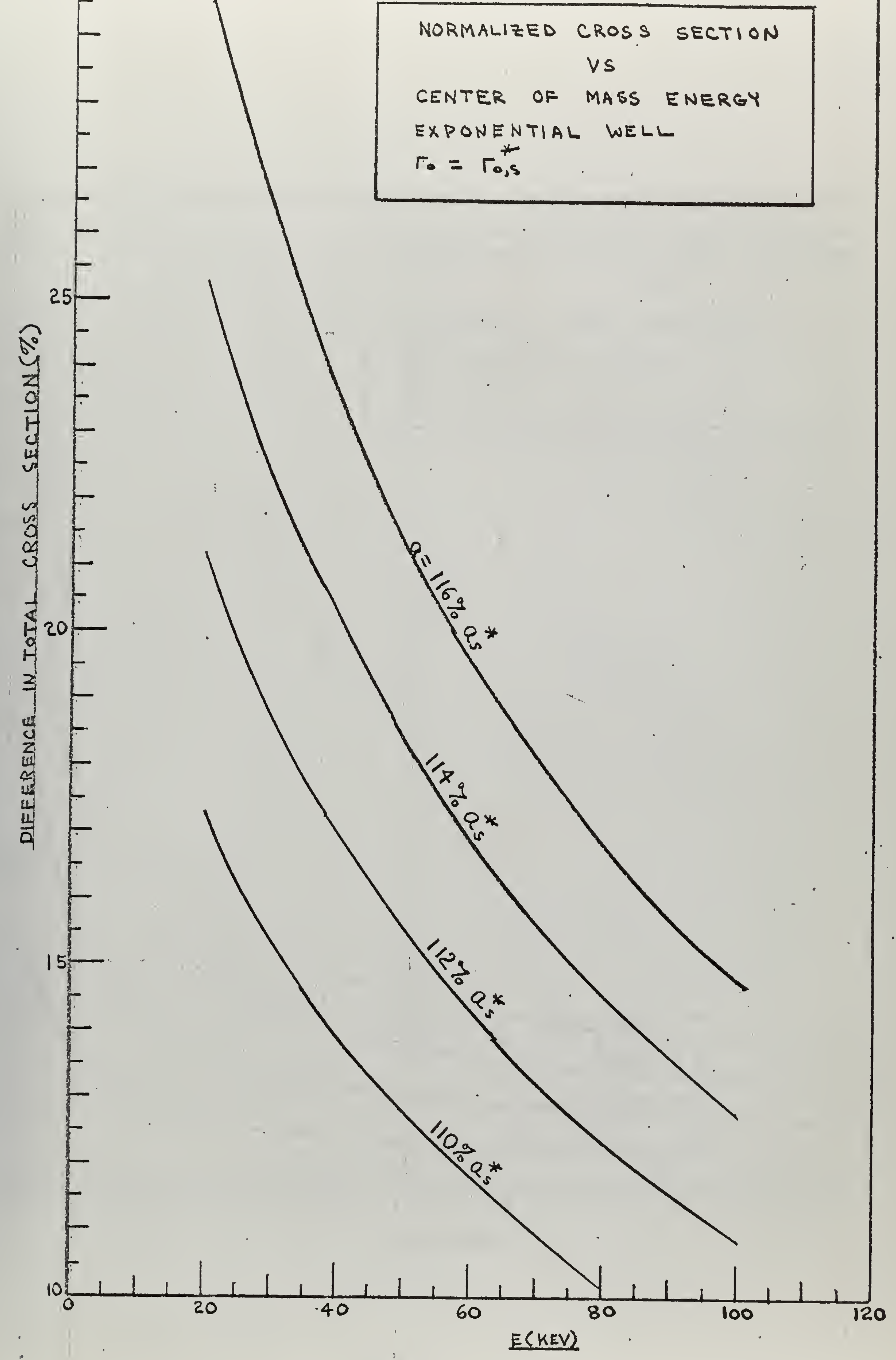
GRAPH 21


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
GAUSSIAN WELL
$r_0 = r_{0,s}^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
15
10
5
0
0
.5
1.0
1.5
2.0
2.5
E (MEV)
$a = 116\% a_s^*$
$112\% a_s^*$
$108\% a_s^*$
$104\% a_s^*$

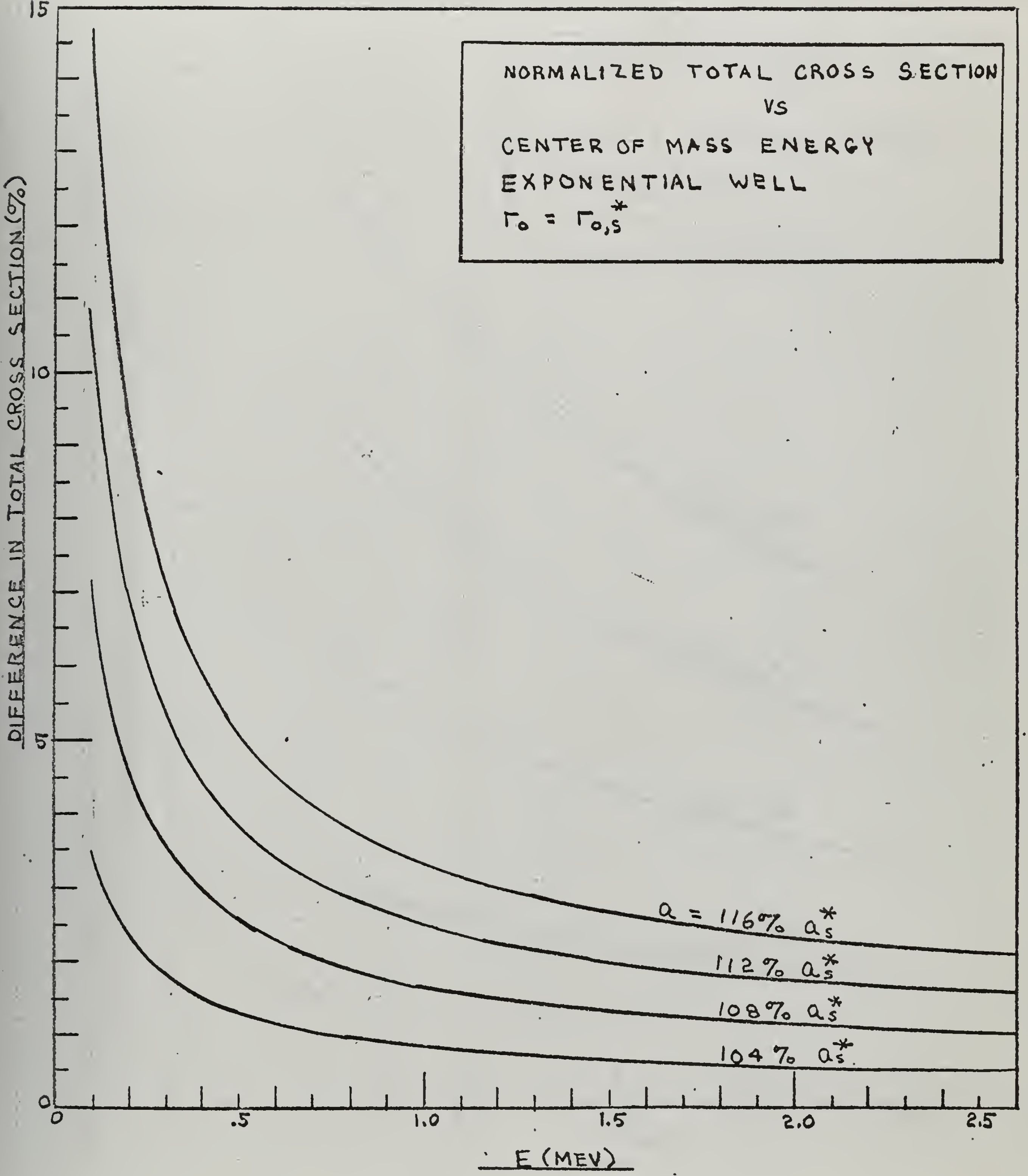
GRAPH 22


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
EXPONENTIAL WELL
$r_0 = r_{0,s}^*$
$a = 108\% \, a_s^*$
$106\% \, a_s^*$
$104\% \, a_s^*$
$102\% \, a_s^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
0
5
10
15
20
40
60
80
100
120
E (KEV)

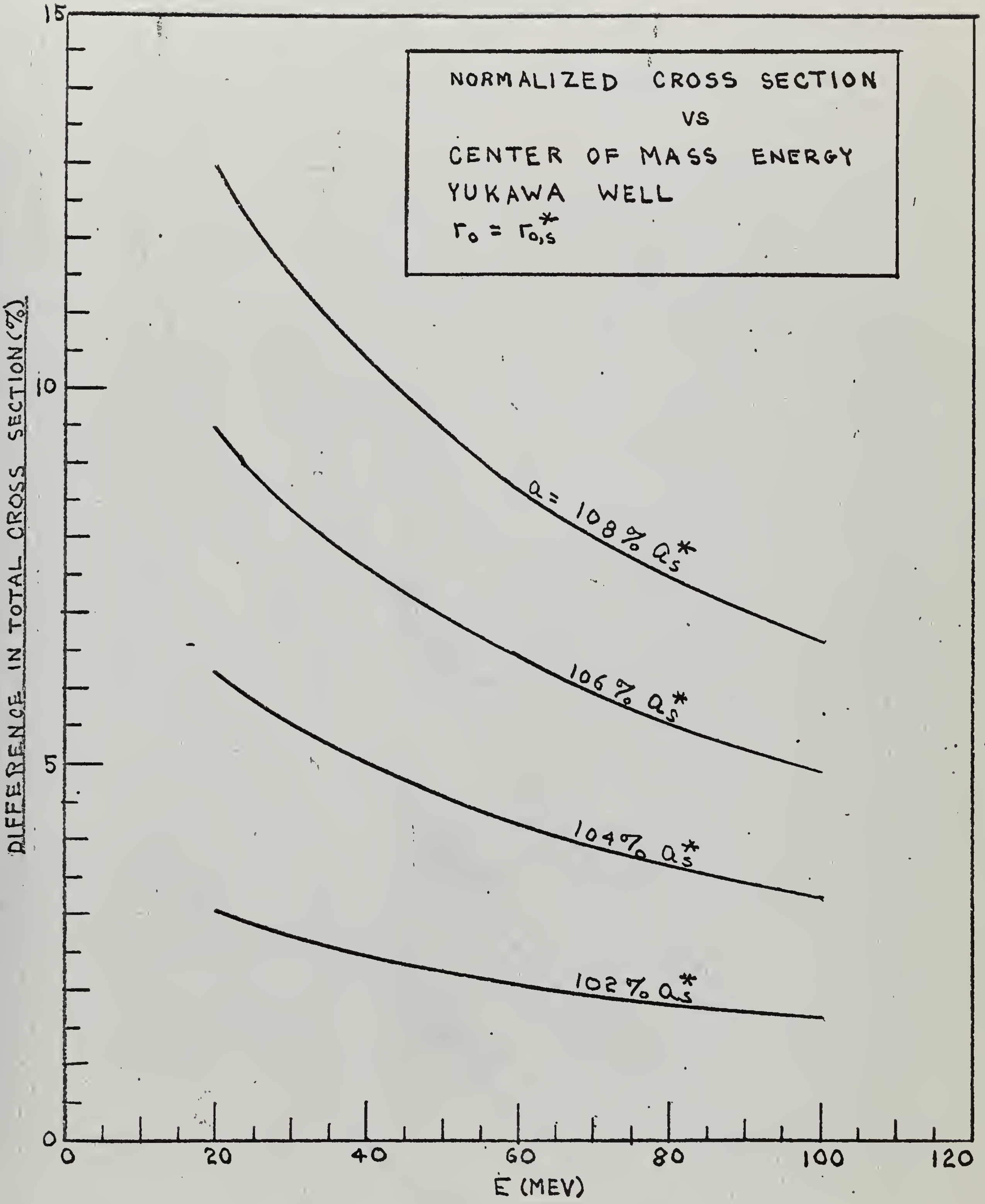
GRAPH 23


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
EXPONENTIAL WELL
$\Gamma_0 = \Gamma_{0,s}^*$
$a = 116\% \, a_s^*$
$114\% \, a_s^*$
$112\% \, a_s^*$
$110\% \, a_s^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
25
20
15
10
0
20
40
60
80
100
120
E(KEV)

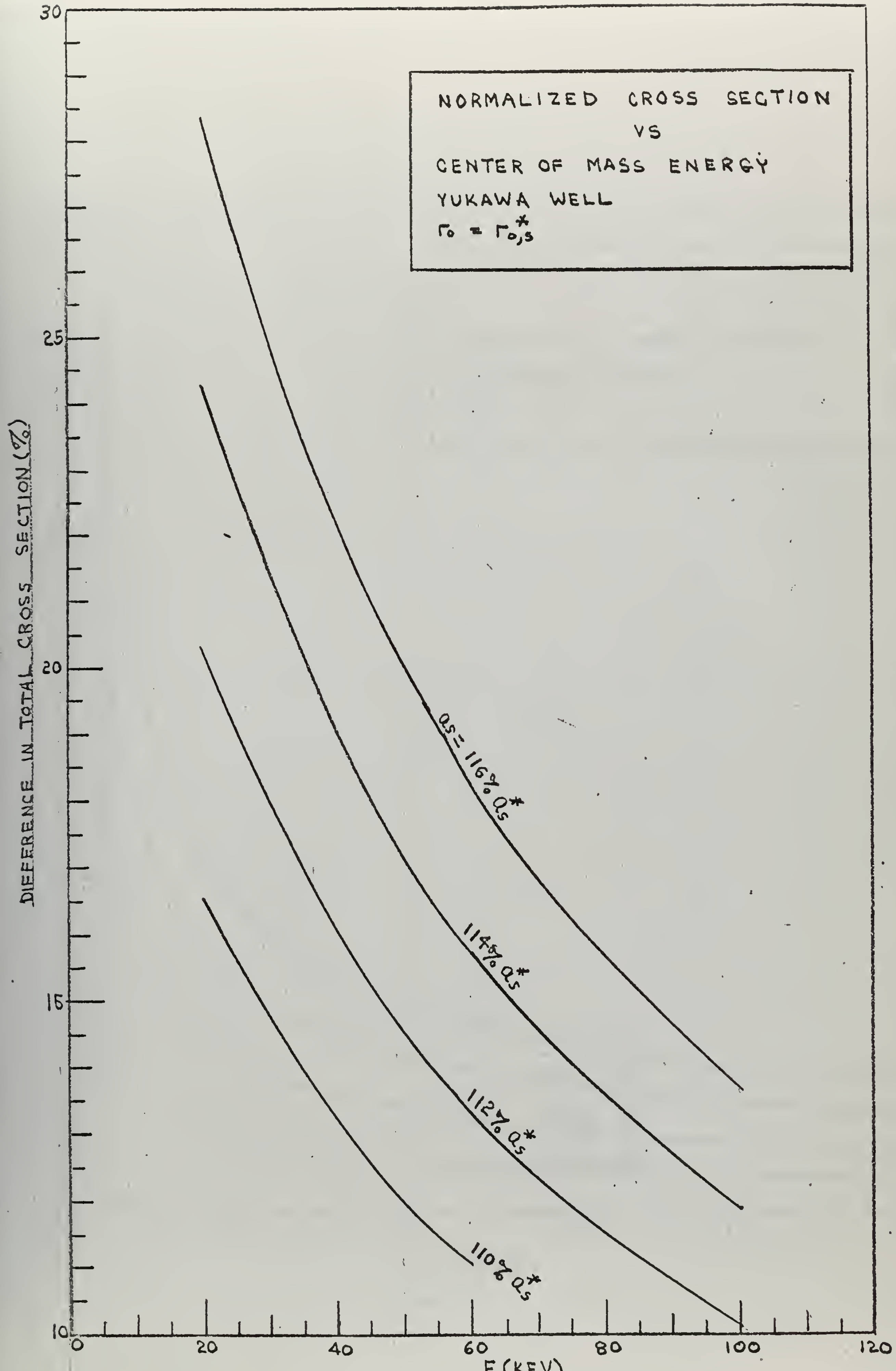
GRAPH 24


NORMALIZED TOTAL CROSS SECTION
VS
CENTER OF MASS ENERGY
EXPONENTIAL WELL
$\Gamma_0 = \Gamma_{0,s}^*$
$a = 116\% \, a_s^*$
$112\% \, a_s^*$
$108\% \, a_s^*$
$104\% \, a_s^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
15
10
5
0
0
.5
1.0
1.5
2.0
2.5
E (MEV)

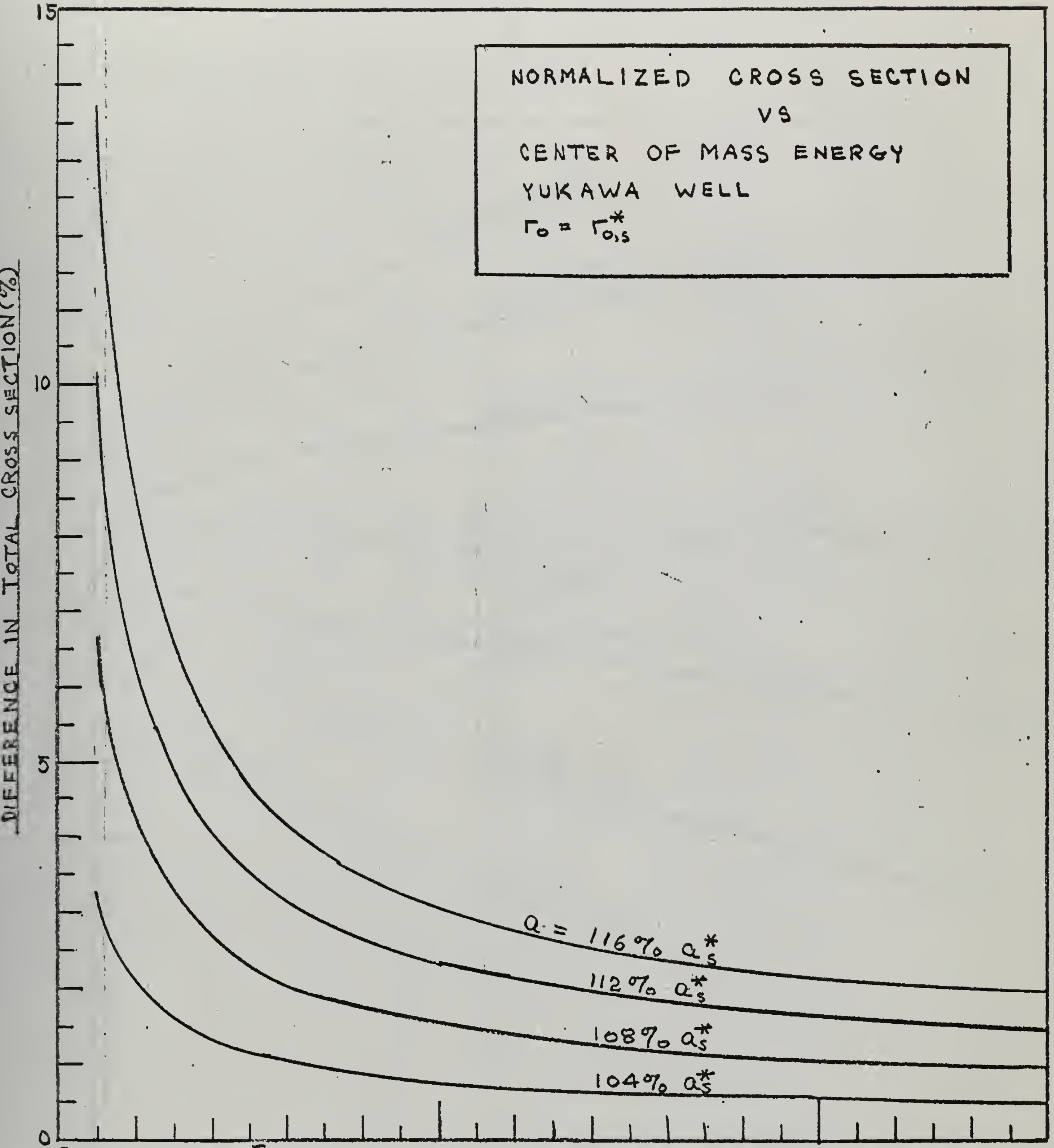
GRAPH 25


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
YUKAWA WELL
$r_0 = r_{0,s}^*$
$a = 108\% \, a_s^*$
$106\% \, a_s^*$
$104\% \, a_s^*$
$102\% \, a_s^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
15
10
5
0
0
20
40
60
80
100
120
E (MEV)

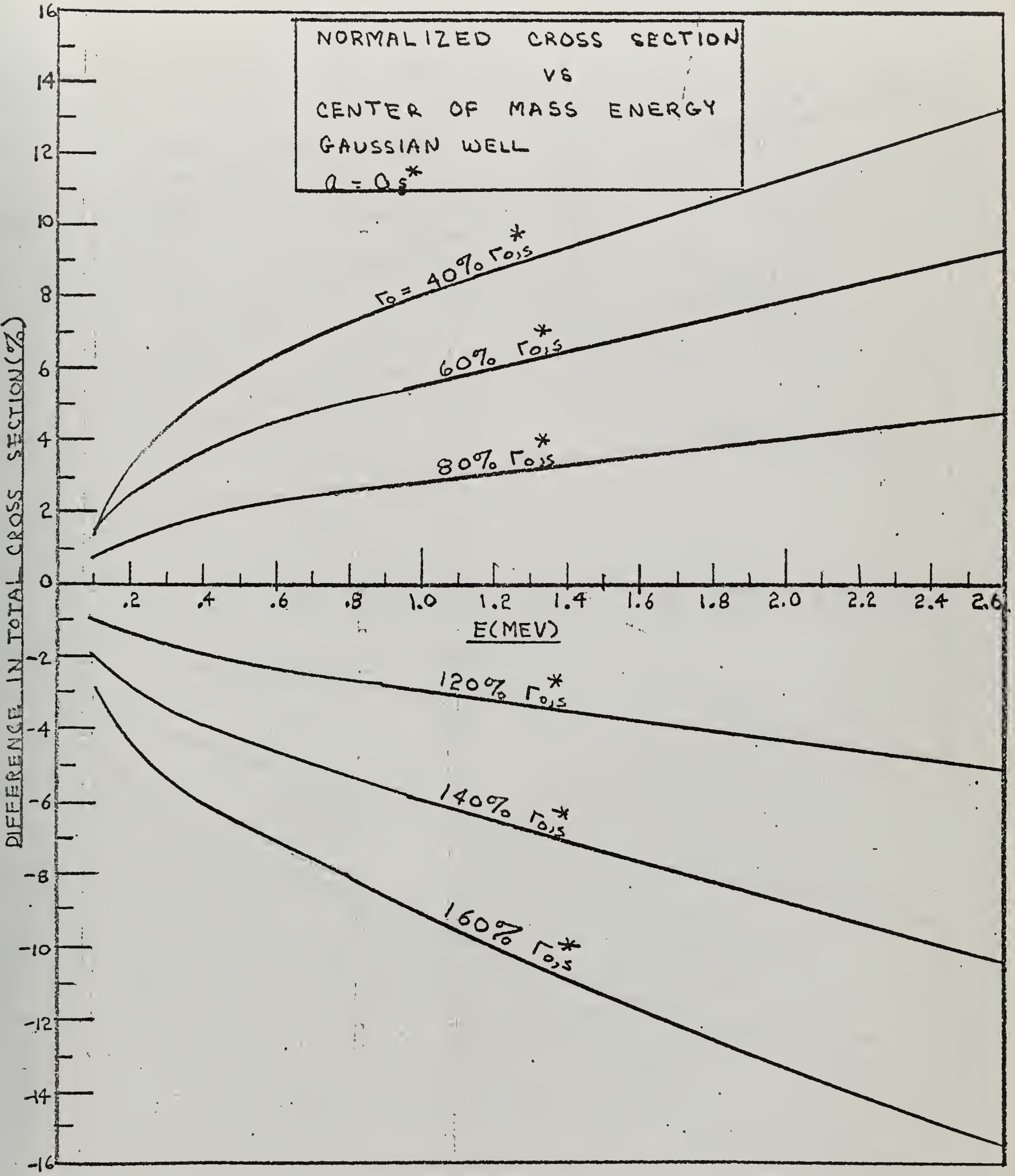
GRAPH 26


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
YUKAWA WELL
$r_0 = r_{0,s}^*$
$a_s = 116\% \, a_s^*$
$114\% \, a_s^*$
$112\% \, a_s^*$
$110\% \, a_s^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
30
25
20
15
10
0
20
40
60
80
100
120
E(KEV)

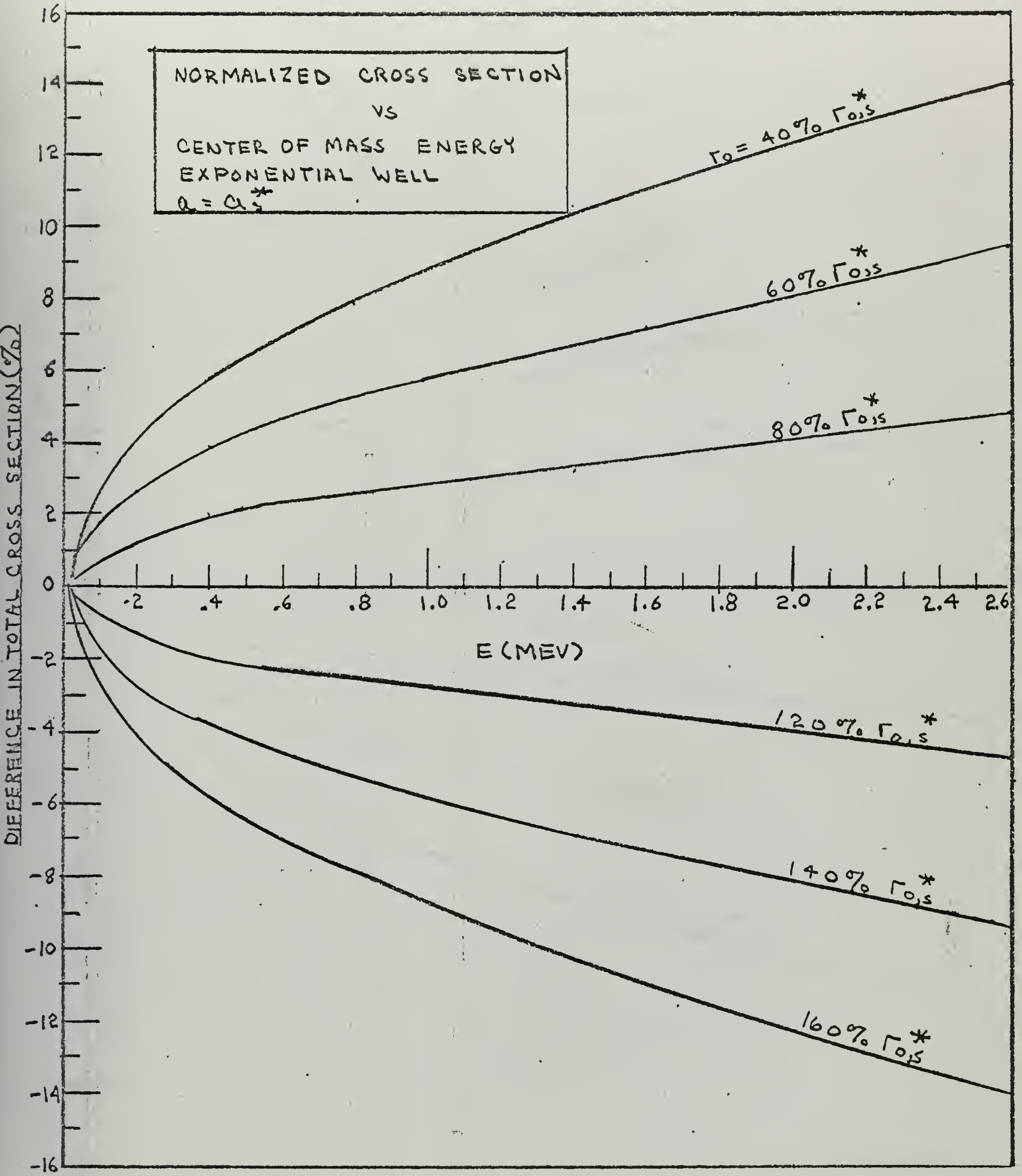
GRAPH 27


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
YUKAWA WELL
$r_0 = r_{0,s}^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
15
10
5
0
0
.5
1.0
1.5
2.0
2.5
E (MEV)
$a = 116\%\ a_s^*$
$112\%\ a_s^*$
$108\%\ a_s^*$
$104\%\ a_s^*$

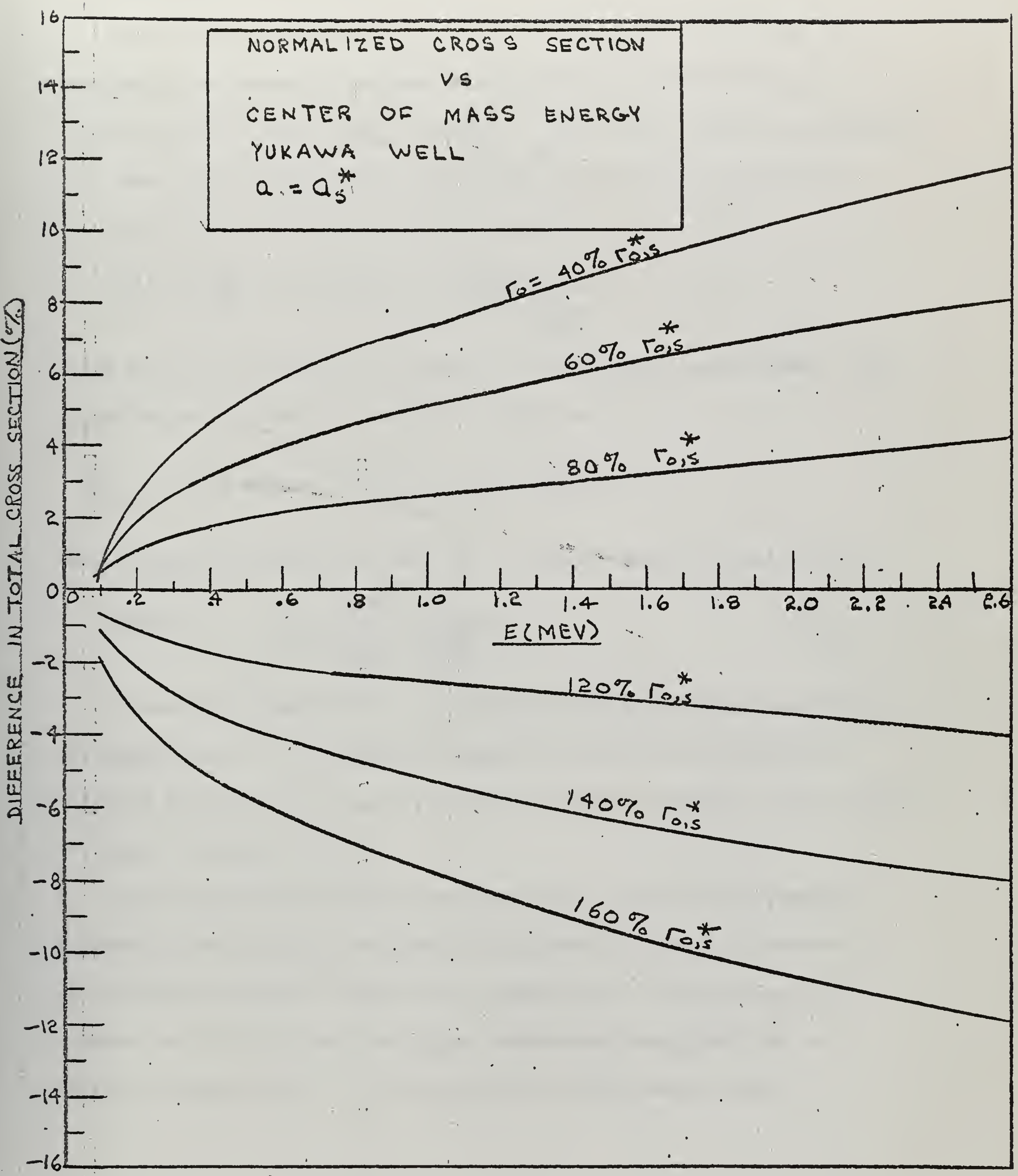
GRAPH 28


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
GAUSSIAN WELL
a = as*
r0 = 40% r0,s*
60% r0,s*
80% r0,s*
120% r0,s*
140% r0,s*
160% r0,s*
DIFFERENCE IN TOTAL CROSS SECTION (%)
E(MEV)
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6

GRAPH 29


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
EXPONENTIAL WELL
$a = a_s^*$
$r_0 = 40\% \, r_{0,s}^*$
$60\% \, r_{0,s}^*$
$80\% \, r_{0,s}^*$
$120\% \, r_{0,s}^*$
$140\% \, r_{0,s}^*$
$160\% \, r_{0,s}^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
E (MEV)
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6

GRAPH 30


NORMALIZED CROSS SECTION
VS
CENTER OF MASS ENERGY
YUKAWA WELL
$a = a_s^*$
DIFFERENCE IN TOTAL CROSS SECTION (%)
E (MEV)
$r_0 = 40\% \ r_{0,s}^*$
$60\% \ r_{0,s}^*$
$80\% \ r_{0,s}^*$
$120\% \ r_{0,s}^*$
$140\% \ r_{0,s}^*$
$160\% \ r_{0,s}^*$
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6

GRAPH 31

5. Shape independent approximation

The total cross section for neutron-neutron scattering was calculated by the shape independent approximation of effective range scattering theory using $r^*_{o,s}$ and a^*_s. This cross section was compared with that from the potential scattering calculations in section 4.

Segré gives for the total cross section

$$\sigma(E) = \frac{4\pi}{k^2}\,\frac{1}{1+\cot^2\delta_0} = \frac{4\pi}{k^2 + \frac{1}{a(k)^2}}$$

where a(k) is the scattering length as a function of wave number. An approximation for a(k) is given for k $\neq$ 0 as:

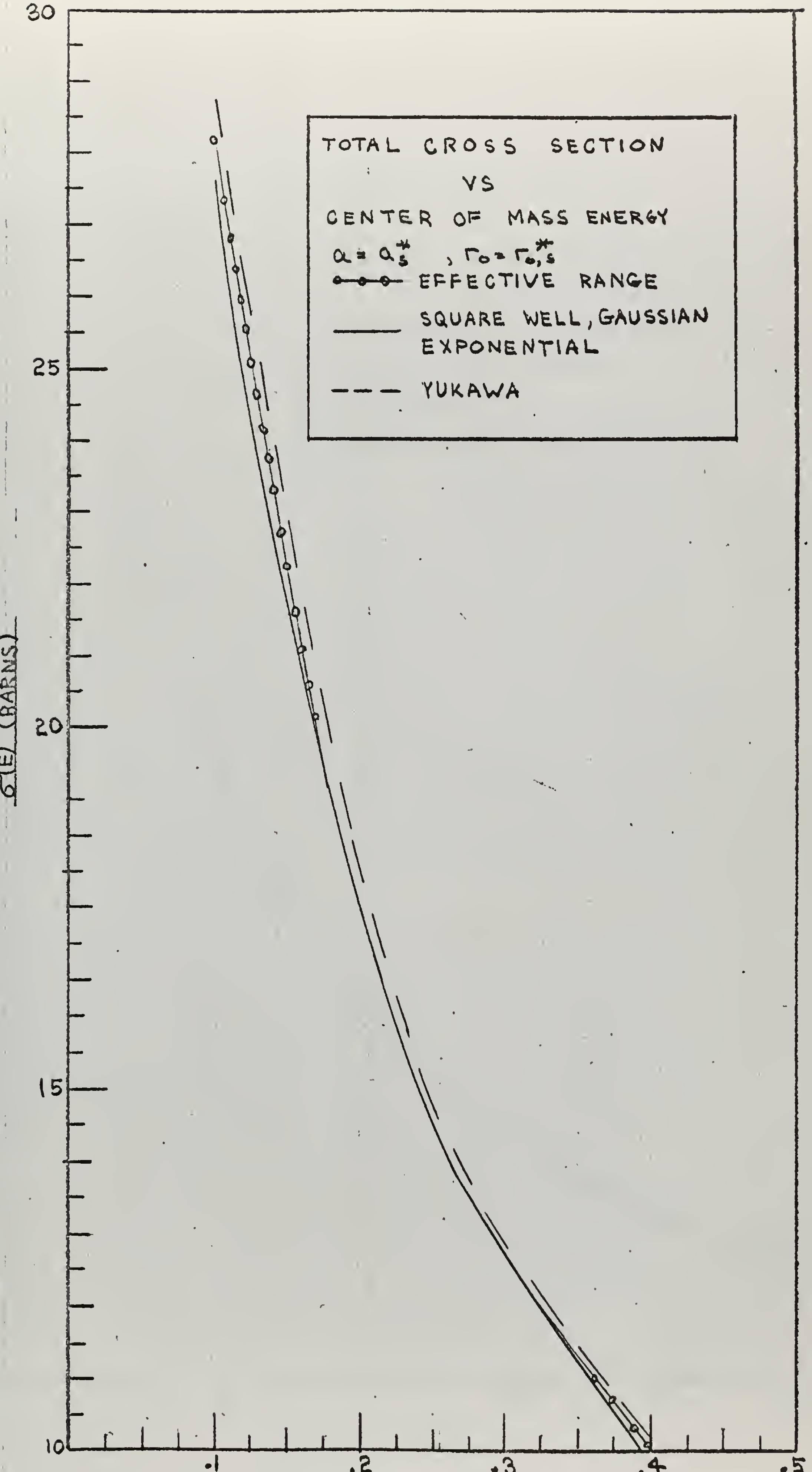
$$1/a(k) = -k\cot\delta_0 = \frac{1}{a(0)} - \frac{1}{2}r_0 k^2 + ---$$

which, when substituted for a(k) in the cross section formula yields:

$$\sigma(E) = \frac{4\pi}{k^2 + \left(\frac{1}{a} - \frac{r_0 k^2}{2}\right)^2} \qquad (1a)$$

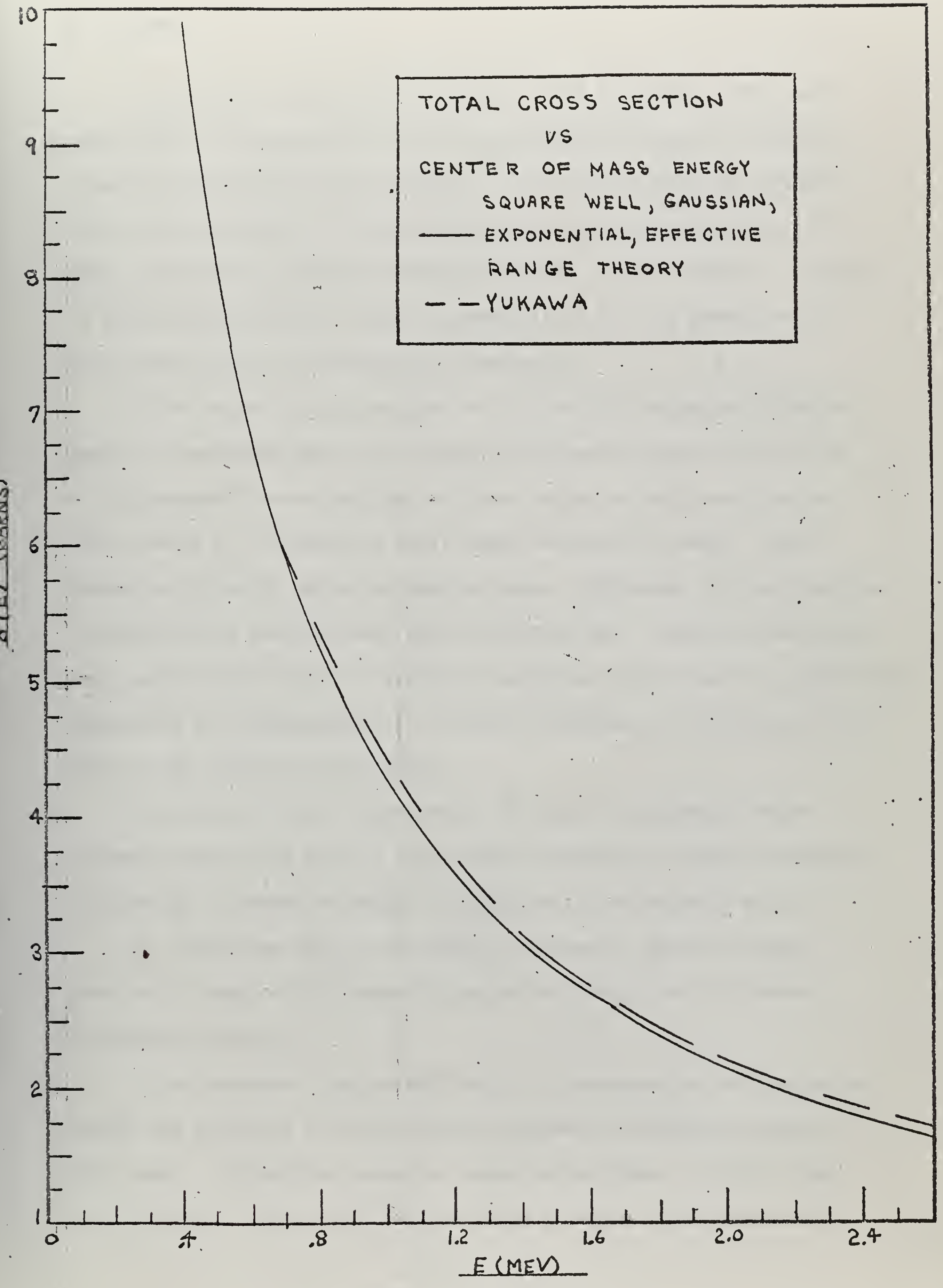
Formula (1a) was used in a calculation of total cross section in the energy range .1 - 2.6MeV and compared with the cross sections obtained from potential scattering for the same parameters. The results are shown in graphs 32 - 33.

The results of the effective range theory calculations compare favorably with potential scattering calculations. A 2% difference is the greatest deviation noted in the comparison. This confirms the commonly held belief that the shape independent approximation of effective range theory is quite accurate in this energy range.



TOTAL CROSS SECTION
VS
CENTER OF MASS ENERGY
$a = a_s^*$, $r_0 = r_{0,s}^*$
EFFECTIVE RANGE
SQUARE WELL, GAUSSIAN EXPONENTIAL
YUKAWA
30
25
20
15
10
.1
.2
.3
.4
.5
σ(E) (BARNS)
E(MEV)

GRAPH 32


TOTAL CROSS SECTION
VS
CENTER OF MASS ENERGY
SQUARE WELL, GAUSSIAN,
EXPONENTIAL, EFFECTIVE
RANGE THEORY
YUKAWA
σ(E) (BARNS)
10
9
8
7
6
5
4
3
2
1
0
.4
.8
1.2
1.6
2.0
2.4
E (MEV)

GRAPH 33

6. Summary

In section 4 it was determined that a 10% scattering experiment could define a scattering length to approximately 6% and an effective range to approximately 40% accuracy. This accuracy might be achieved with only two values of cross section; that at 20 kev and that at 2.6 MeV, if the energy could be determined to 10% in both regions. Certainly it would be achieved by a least squares fit at 10 - 20 appropriately distributed energies as proposed by Moravcsik.

In the region of the accepted n-p scattering length and effective range, it was found that a 2% change in scattering length resulted in a 3.1% change in cross section at 20 kev center of mass energy while only causing a .1% change in total cross section at 2.6MeV. A 20% change in effective range was seen to cause a 5% change in cross section at 2.6MeV while having almost no effect at 20 kev. Thus the conclusion was reached that effective range and scattering length could be determined separately and independently by the cross sections at the high and low ends of the proposed energy range.

In section 3 it was found that a 1% change in potential depth or range resulted in about a 20% change in scattering length and about a 1% change in effective range for Exponential and Gaussian wells.

For the Yukawa well, a 1% change in potential depth and range resulted in about a 12% change in scattering length and a 1% change in effective range.

It was conversely determined that a 6% determination of scattering length can determine the Gaussian and Exponential potential depths to less than - .2% and the potential range to less than - .3% for the n-p scattering length and effective range vicinity. The Yukawa well

depth would be determined to less than -.5% and the potential range to -.2%.

It therefore seems quite likely that a 10% scattering experiment would determine the scattering length and effective range accurately enough to furnish an acceptable set of potential parameters. These potential parameters appear to be adequate to investigate charge-independent and charge-symmetry effects.

Effective range theory was quite adequate to determine cross sections in the examined energy range. A 1% agreement was found with potential scattering. There should be little question of its propriety here.

ACKNOWLEDGMENT

The patient aid and excellent counsel provided by Professor E. B. Buskirk is gratefully acknowledged and greatly appreciated.

APPENDIX I

COMPUTER PROGRAM DESCRIPTION

The program is written in FORTRAN II language and, as listed in Appendix III, is arranged to operate with the Control Data Corporation-1604 as a central processor and the I B M - 1401 as peripheral equipment. The library tape functions used are the standard FORTRAN II functions with the exception of a subroutine called DRAW. This subroutine is a part of the library tape at the Computer Center of the United States Naval Postgraduate School and is identified by its large computer operators co-op ID: J7-NPS-DRAW. The subroutine was written by J.R. Ward of the U.S. Naval Postgraduate School. Where this subroutine is not available, the program can be run by omitting subroutines SKETCH and NOW and their call cards.

The program consists of a main routine and 13 subroutines. The main routine calls the subroutines in proper sequence and also provides for a sequential variation of the parameters r_n, and V_o. The subroutines and the function each performs are listed as follows:

Subroutine INPUT2	Reads in data cards
Subroutine READY2	Calculates and tabulates energies, wave numbers, scattering lengths, effective ranges, potential parameters, and calls bessel functions.
Subroutine FIGURE	Prepares for and calls integration routine, forms the logrithmic derivative of the integrated function.
Subroutine RKUTTA	Integrates equation provided by DERIV for $\ell = 0$.
Subroutine RKUTTA1	Integrates equation provided by DERIV1 for $\ell = 1$.
Subroutine DERIV	Provides $\ell = 0$ equation for integration for a particular potential.

Subroutine DERIV1	Provides $\ell = 1$ equation for integration for a particular potential.
Subroutine FINISH	Computes phase shifts, cross sections, and normalized cross sections.
Subroutine OUTPUT	Prints out resulting r_o/a ratio, V_o, r_n, energy, phase shifts, and cross sections.
Subroutine BESSEL	Calculates bessel functions
Subroutine SKETCH	Prepares for and calls DRAW to plot all cross sections on one graph for comparison.
Subroutine APPEND	Stores base cross section for normalization in FINISH.
Subroutine NOW	Prepares for and calls DRAW to plot all normalized cross sections on one graph for comparison.

APPENDIX II

PROGRAM DICTIONARY

EINT	Initial value of center of mass energy in MeV.
EINC	Incremental value of center of mass energy in MeV.
II	Maximum number of center of mass energy values.
ROS	Base effective range in cm.
AS	Base scattering length in cm.
CHANGE	Percentage incremental change in base effective range or base scattering length in decimal form.
MANY	Number of + or - incremental charges to be made.
REND	Radius at which integrated solution is matched to exact solution for phase shift determination in cm.
NUMBER	Number of integration steps.
MODE	Print out control for program monitoring = 0 for result print out only = 1 for minor progress report = 2 for print out of all calculations
III	Energy subscript for wave function print out.
MOMENTA	= 0 $\ell = 0$ calculations only = 1 $\ell = 1$ calculations only = 2 both $\ell = 0$ and $\ell = 1$ calculations.
FK	Wave number, k
FK2	Wave number squared, k^2.
FMANY	MANY in floating point.
RA	Ratio of effective range to scattering length, r_o/a.
S	Well depth parameter, s

B	Intrinsic range, b
POT	Well depth, V_o
RN	Well range, r_n
ARGB	Bessel function argument, kr
VALUE	Output array of bessel functions
VALJO	Spherical $\ell = 0$ bessel function, j
VALJ1	Spherical $\ell = 1$ bessel function, j_1
VALJ2	Spherical $\ell = 2$ bessel function, j_2
VALNO	Spherical $\ell = 0$ neumman function, n_0
VALN1	Spherical $\ell = 1$ neumann function, n_1
VALN2	Spherical $\ell = 2$ neumann function, n_2
A	A particular potential range from array, RN
V	A particular potential depth from array, POT
FNUMBER	Number of integration steps in floating point
STEP	integration step size
R	Array of radius values at each integration step
YVARS(1)	Wave function
YVARS(2)	Slope of wave functions
XVAR	Radius variable of integration
Y	Wave function at end value of integration step for $\ell = 0$.
YDOT	Slope of wave function at end value of integration step for $\ell = 0$.
COMPARO	Logrithmic derivative of wave function at end of integration range for $\ell = 0$.
COMPAR1	Logrithmic derivative of wave function at end of integration range for $\ell = 1$.
XC	Runge kutta weighted radius step

YC	Dependent variable for XC
DY	Derivative of dependent variable for XC
YL	Same as Y for $\ell = 1$.
YLDOT	Same as YDOT for $\ell = 1$.
DELTA0	$\ell = 0$ phase shift.
DELTA1	$\ell = 1$ phase shift.
XSEC0	$\ell = 0$ cross section
XSEC1	$\ell = 1$ cross section
TOTAL	Sum of singlet and triplet cross sections
BETTER	Normalized $\ell = 0$ cross sections
FNORM	Base cross section to be used for normalization
NUMBERR	Number of simultaneous first order differential equations to be integrated.

APPENDIX III
PROGRAM LISTING

```
  MAIN PROGRAM
  DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
 1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
 2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
 300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
 40),FNORM(100),BETTER(10,100)
  COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
 1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
 2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
 3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
 4TTER
  CALL INPUT2
  CALL READY2
  A=RN(MANY+1)
  V=POT(MANY+1)/.41499E-24
  CALL FIGURE
  CALL FINISH
  CALL APPEND
  DO 1 L=1,LL
  A=RN(L)
  V= POT(L)/.41499E-24
  CALL FIGURE
  CALL FINISH
  CALL OUTPUT2
  CALL SKETCH(E,XSEC0,MOD,NUMPTS,II,L,LL)
1 CONTINUE
  CALL NOW(E,BETTER,II,L,LL)
  END
```

```
      SUBROUTINE INPUT2
      DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
     1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
     2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
     300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
     40),FNORM(100),BETTER(10,100)
      COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
     1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
     2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
     3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
     4TTER
      READ 1,EINT,EINC,II
    1 FORMAT(2E20.4,I10)
      READ 2,ROS,AS,CHANGE,MANY
    2 FORMAT(2E20.6,F10.4,I10)
      READ 3,REND,NUMBER
    3 FORMAT(E20.4,I10)
      READ 4,MODE
    4 FORMAT(I10)
      READ 5,III
    5 FORMAT(I10)
      READ 6,MOMENTA
    6 FORMAT(I10)
      IF(MODE - 1) 8,8,7
    7 PRINT 1,EINT,EINC,II
      PRINT 2,ROS,AS,CHANGE,MANY
      PRINT 3,REND,NUMBER
      PRINT 4,MODE
      PRINT 5,III
      PRINT 6,MOMENTA
    8 RETURN
      END
```

```
    SUBROUTINE FIGURE
    DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
   1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
   2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
   300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
   40),FNORM(100),BETTER(10,100)
    DIMENSION YVARS(30)
    COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
   1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
   2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
   3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
   4TTER
    FNUMBER = NUMBER
    STEP = REND / FNUMBER
    R(1) = 0.
    NN = NUMBER + 1
    DO 299 N=2,NN
299 R(N) = R(N-1) + STEP
    DO 292 K=2,II
    YVARS(1) = 0.
    YVARS(2) = 1.
    IF(MOMENTA - 1) 283,284,283
283 CALL RKUTTA(2,XVAR,YVARS,STEP)
    IF(K-III) 293,297,293
297 PRINT 296
296 FORMAT (13X,7H ENERGY,13X,7H RADIUS,6X,14H WAVE FUNCTION)
    PRINT 295,E(K)
295 FORMAT(F20.2)
    PRINT 294, (R(N), Y(N), N=1,NN)
294 FORMAT (E40.4,E20.4)
293 COMPAR0(K)=YDOT(NN-1)/Y(NN-1)
284 IF (MOMENTA - 1) 292,291,291
291 YVARS(1) = 0.
    YVARS(2) = 1.
    CALL RKUTTA1(2,XVAR,YVARS,STEP)
    IF(K-III) 285,289,285
289 PRINT 288
288 FORMAT (13X,7H ENERGY,13X,7H RADIUS,6X,18H WAVE FUNCTION L=1)
    PRINT 287,E(K)
287 FORMAT(F20.2)
    PRINT 286, (R(N),YL(N),N=1,NN)
286 FORMAT(E40.4,E20.4)
285 COMPAR1(K)=YLDOT(NN-1)/YL(NN-1)
292 CONTINUE
    RETURN
    END
```

```
      SUBROUTINE  RKUTTA(NUMBERR,XVAR,YVARS,STEP)
      DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
     1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
     2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
     300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
     40),FNORM(100),BETTER(10,100)
      DIMENSION YVARS(30),AK(4,30),DY(30),YC(30),C(4)
      COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
     1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
     2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
     3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
     4TTER
      C(1)=0.0
      C(2)=0.5
      C(3)=0.5
      C(4)=1.0
      DO 298 N=1,NN
      XVAR  = R(N)
      DO 1 I=1,4
      XC=XVAR + C(I)*STEP
      DO 2 J=1,NUMBERR
    2 YC(J) =YVARS(J) + C(I)*AK(I-1,J)
      CALL DERIV(XC,YC,DY)
      DO 1 J=1,NUMBERR
    1 AK(I,J) = STEP * DY(J)
      DO 3 J = 1,NUMBERR
    3 YVARS(J) =YVARS(J) + (AK(1,J)+2.*AK(2,J)+2.*AK(3,J)+AK(4,J))/6.
      YDOT(N) = YVARS(2)
  298 Y(N) = YVARS(1)
      RETURN
      END
```

```
      SUBROUTINE RKUTTA1(NUMBERR,XVAR,YVARS,STEP)
      DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
     1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
     2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
     300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(1C
     40),FNORM(100),BETTER(10,100)
      DIMENSION YVARS(30),AK(4,30),DY(30),YC(30),C(4)
      COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
     1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
     2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
     3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
     4TTER
      C(1)=0.0
      C(2)=0.5
      C(3)=0.5
      C(4)=1.0
      DO 290 N=1,NN
      XVAR = R(N)
      DO 1 I=1,4
      XC=XVAR + C(I)*STEP
      DO 2 J=1,NUMBERR
    2 YC(J) =YVARS(J) + C(I)*AK(I-1,J)
      CALL DERIV1(XC,YC,DY)
      DO 1 J=1,NUMBERR
    1 AK(I,J) = STEP * DY(J)
      DO 3 J = 1,NUMBERR
    3 YVARS(J) =YVARS(J) + (AK(1,J)+2.*AK(2,J)+2.*AK(3,J)+AK(4,J))/6.
      YL(N) = YVARS(1)
  290 YLDOT(N) = YVARS(2)
      RETURN
      END
```

```
      SUBROUTINE FINISH
      DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
     1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
     2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
     300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
     40),FNORM(100),BETTER(10,100)
      COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
     1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
     2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
     3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
     4TTER
      IF(MOMENTA - 1) 399,397,399
  399 DO 398 I=2,II
      DELTA0(I)=ATANF((REND*(COMPAR0(I)*VALJ0(I)+FK(I)*VALJ1(I))-
     1VALJ0(I))/(REND*(COMPAR0(I)*VALN0(I)+FK(I)*VALN1(I))-
     2VALN0(I)))
  398 XSEC0(I)=4.*3.1416/FK2(I)*SINF(DELTA0(I))**2
  397 IF(MOMENTA-1)394,396,396
  396 DO 395 I=2,II
      DELTA1(I)=ATANF((REND*(FK(I)/3.*VALJ0(I)-2./3.*FK(I)*
     1VALJ2(I)-COMPAR1(I)*VALJ1(I))+VALJ1(I))/(REND*(FK(I)/3.*
     2VALN0(I)-2./3.*FK(I)*VALN2(I)-COMPAR1(I)*VALN1(I))+VALN1(I)))
  395 XSEC1(I)=12.*3.1416/FK2(I)*SINF(DELTA1(I))**2
  390 DO 389 I = 2,II
  389 TOTAL(I) = .25 * XSEC0(I) + .75 * XSEC1(I)
  394 DO 900 I=2,II
      BETTER(L,I) = XSEC0(I)/FNORM(I)
  900 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OUTPUT2
      DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
     1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
     2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
     300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
     40),FNORM(100),BETTER(10,100)
      COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
     1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
     2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
     3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
     4TTER
      PRINT 600,RA(L)
  600 FORMAT(34H EFFECTIVE RANGE/SCATTERING LENGTH,/,E34.10)
      PRINT 599,A,POT(L)
  599 FORMAT (13X,7H RADIUS,4X,16H POTENTIAL DEPTH,/,E20.4,E20.4)
      PRINT 598
  598 FORMAT(7H ENERGY,12H PHASE ANGLE,15H CROSS SECTION,12H PHASE ANGLE
     1,15H CROSS SECTION,2X,6H TOTAL,/,11X,4H L=0,7X,4H L=0,10X,4H L=1,
     28X,4H L=1,8X,6H CROSS,/,63X,8H SECTION)
      PRINT 597,(E(I),DELTA0(I),XSEC0(I),DELTA1(I),XSEC1(I),TOTAL(I),I=2
     1,II)
  597 FORMAT(F7.2,E12.4,E15.4,E12.4,E15.4,E11.4)
      PRINT 596
  596 FORMAT(7H ENERGY,5X,25H NORMALIZED CROSS SECTION)
      PRINT 595,(E(I),BETTER(L,I),I=2,II)
  595 FORMAT(F7.2,E20.8)
      RETURN
      END
```

```
    SUBROUTINE BESSEL (X,N,VALUE,INDEX,MM)
    DIMENSION VALUE(40),BJ(100),F(100)
    IF (X) 10,10,9
  9 IK=X/2.0
    K=N+20+IK
    IF(K-100) 300,299,299
299 PRINT 298
298 FORMAT(66H ARGUMENT PLUS ORDER TOO LARGE. INCREASE DIMENSIONS AND
   1RUN AGAIN.)
    RETURN
300 CONTINUE
    Z=1.0/X
    PI=3.141592653
    GOTO(40,40,60,60,80,90,100,110),INDEX
 10 PRINT 8,X
  80FORMAT (1X,63HSUBROUTINE BESSEL DOES NOT WORK FOR X=0 OR LESS THAN
   1 0, HERE X= E15.8)
    RETURN
 40 BJ(K+1)=0.0
    BJ(K)=10.0E-30
    L=K
 11 FL=L
    BJ(L-1)=2.0*FL*Z*BJ(L)-BJ(L+1)
    IF(L-1) 12,12,13
 13 L=L-1
    GO TO 11
 12 SUM=0.0
    IK=K
    DO 14 I=2,IK,2
 14 SUM=SUM+BJ(I)
    C=1.0/(BJ(0)+2.0*SUM)
    GO TO(42,43),INDEX
 43 DO 45 I=0,K
 45 BJ(I)=C*BJ(I)
    GO TO 54
 42 DO 41 I=0,N
 41 VALUE(I)=C*BJ(I)
    RETURN
 50 GO TO 40
 54 EC=.5772156649
    SUM=0.0
    IK=K/2
    DO 15 I=1,IK
    FI=I
 15 SUM=SUM+((-1.0)**(I-1)*BJ(2*I))/FI
    F(0)=(2.0/PI)*(BJ(0)*(LOGF(X/2.0)+EC)+2.0*SUM)
    F(1)=(BJ(1)*F(0)-2.0/(PI*X))/BJ(0)
    IN=N-1
    DO 16 I=1,IN
    FI=I
```

```
  16 F(I+1)=2.0*FI*Z*F(I)-F(I-1)
     DO 51 I=0,N
  51 VALUE(I)=F(I)
     RETURN
  60 BJ(K+1)=0.
     BJ(K)=10.E-30
     L=K
  19 FL=L
     BJ(L-1)=2.*FL*Z*BJ(L)+BJ(L+1)
     IF(L-1)17,17,18
  18 L=L-1
     GO TO 19
  17 SUM=0.0
     DO 21 I=1,K
  21 SUM=SUM+BJ(I)
     C=(EXPF(X))/(BJ(0)+2.*SUM)
     GO TO(1,1,61,62),INDEX
   1 PRINT 1000
     RETURN
1000 FORMAT(40H CARD MISSING. IGNORE RESULTS.
  62 DO 64 I=0,1
  64 BJ(I)=C*BJ(I)
     GO TO 71
  61 DO 65 I=0,N
  65 VALUE(I)=C*BJ(I)
     RETURN
  70 GO TO 60
  71 IF(5.0-X) 120,121,121
 121 M=10
     GO TO 122
 120 M=5
 122 F (0)=GAUSS(0.0,1.0,M,X)
     F(1)=(1./X-F(0)*BJ(1))/BJ(0)
     IN=N-1
     DO 22 I=1,IN
     FI=I
  22 F (I+1)=2.0*FI*Z*F (I)+F (I-1)
     DO 72 I=0,N
  72 VALUE(I)=F (I)
     RETURN
  80 F(K+1)=0.0
     F (K)=10.0E-30
     L=K
  25 FL=L
     F (L-1)=(2.0*FL+1.0)*Z*F (L)-F (L+1)
     IF (L-1)  23,23,24
  24 L=L-1
     GO TO 25
  23 C=SINF(X)/(X*F (0))
     IF(MM)200,200,201
 201 SQ=SQRTF(2.*X/PI)
     DO 202 I=0,N
 202 VALUE(I)=C*SQ*F (I)
     RETURN
```

```
  200 DO 26 I=0,N
   26 VALUE(I)=C*F (I)
      RETURN
   90 F (0)=-COSF(X)/X
      F (1)=-SINF(X)/X-COSF(X)/X**2
      IN=N-1
      DO 27 I=1,IN
      FI=I
   27 F(I+1)=(2.*FI+1.0)*Z*F(I)-F(I-1)
      IF(MM) 203,203,204
  204 SQ=SQRTF(2.*X/PI)
      DO 205 I=0,N
  205 VALUE(I)=F (I)*SQ*(-1.)**(-I-1)
      RETURN
  203 DO 91 I=0,N
   91 VALUE(I)=F (I)
      RETURN
  100 F (K+1)=0.0
      F (K)=10.0E-30
      L=K
   30 FL=L
      F (L-1)=(2.0*FL+1.0)*Z*F (L)+F (L+1)
      IF (L-1) 28,28,29
   29 L=L-1
      GO TO 30
   28 C=SINH (X)/(X*F (0))
      IF(MM) 206,206,207
  207 SQ=SQRTF(2.*X/PI)
      DO 208 I=0,N
  208 VALUE(I)=C*SQ*F (I)
      RETURN
  206 DO 101 I=0,N
  101 VALUE(I)=C*F (I)
      RETURN
  110 F (0)=-PI*EXPF(-X)/(2.0*X)
      F (1)=(PI*(X+1.0)*EXPF(-X))/(2.0*X**2)
      IN=N-1
      DO 31 I=1,IN
      FI=I
   31 F (I+1)=F (I-1)-(2.0*FI+1.0)*Z*F (I)
      IF(MM) 209,209,210
  210 SQ=SQRTF(2.*X/PI)
      DO 211 I=0,N
  211 VALUE(I)=F (I)*SQ*(-1.)**(-I-1)
      RETURN
  209 DO 111 I=0,N
  111 VALUE(I)=F (I)
      RETURN
      END
C       FUNCTION GAUSS  NUMERICAL INTERGRATION
C       FUNCTION GAUSS REQUIRES FUNCTION GAUSSF(X) NUMERICAL INTERGRATION
      FUNCTION GAUSS(G,B,M,X)
      DIMENSION R(10),U(10)
      U(1)=+.0744371695
```

```
      U(2)=-.0744371695
      U(3)=+.2166976971
      U(4)=-.2166976971
      U(5)=+.3397047841
      U(6)=-.3397047841
      U(7)=+.4325316833
      U(8)=-.4325316833
      U(9)=+.4869532643
      U(10)=-.4869532643
      R(1)=+.1477621124
      R(2)=+.1477621124
      R(3)=+.1346333597
      R(4)=+.1346333597
      R(5)=.10954318113
      R(6)=.10954318113
      R(7)=+.07472567458
      R(8)=+.07472567458
      R(9)=+.03333567215
      R(10)=+.03333567215
      A=G
      FN=M
      P=(B-A)/FN
      GAUSS=0.0
      DO 1 J=1,M
      C=0.0
      Y=A+P
      DO 2 I=1,10
      D=(R(I)*GAUSSF((Y-A)*U(I)+(A+Y)/2.0,X))*(Y-A)
    2 C=D+C
      A=Y
    1 GAUSS=GAUSS +C
      RETURN
      END
      FUNCTION GAUSSF(U,X)
      IF(1.0/U-710.0)123,124,124
  124 GAUSSF=0.0
      RETURN
  123 GAUSSF= (EXPF(-X*COSH(1.0/U-1.0)))/U**2
      END
```

```
  SUBROUTINE SKETCH(E,XSECO,MOD,NUMPTS,II,L,LL)
  DIMENSION E(100),XSECO(100),XSEC1(100),TOTAL(100)
  DIMENSION X(900),Y(900),ITITLE(12)
  LABEL = 4H
  DO 1 I=1,12
1 ITITLE(I) = 8H
  ITITLE(1)=8HOSGOOD,F
  ITITLE(2)=8H.B.,BOX
  ITITLE(3)=8H09
  ITITLE(4)=8HEXPONENT
  ITITLE(5)=8HIAL WELL
  ITITLE(6)=8H RUN 15
  ITITLE(7)=8H GRAPH O
  ITITLE(8)=8HF CROSS
  ITITLE(9)=8HSECTION
  ITITLE(10)=8HVS ENERG
  ITITLE(11)=8HY FOR L=
  ITITLE(12) = 8H0
  NUMPTS = II-1
  IF(L-1)2,2,3
2 MOD = 1
  GO TO 6
3 IF(LL-L) 5,5,4
4 MOD = 2
  GO TO 6
5 MOD = 3
6 CONTINUE
  CALL DRAW(NUMPTS,E,XSECO,MOD,0,LABEL,ITITLE,.3,0,0,0,0,0,9,15,1,L
 1AST)
  RETURN
  END
```

```
      SUBROUTINE APPEND
      DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
     1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
     2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
     300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
     40),FNORM(100),BETTER(10,100)
      COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
     1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
     2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
     3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
     4TTER
      DO 1 I=2,II
    1 FNORM(I) = XSEC0(I)
      RETURN
      END
```

```
      SUBROUTINE NOW(E,BETTER,II,L,LL)
      DIMENSION E(100), BETTER(10,100),X(900),Y(900),ITITLE(12)
      LABEL = 4H
      DO 1 I=1,12
 1    ITITLE(I)=8H
      ITITLE(1)=8HOSGOOD,F
      ITITLE(2)=8H.B.,BOX
      ITITLE(3)=8H09
      ITITLE(7)=8HNORMALIZ
      ITITLE(8)=8HED CROSS
      ITITLE(9)=8H SECTION
      ITITLE(10)=8H FOR L=0
      NUMPTS = II-1
      DO 7 L=1,LL
      IF(L-1) 2,2,3
 2    MOD=1
      GO TO 6
 3    IF(LL-L) 5,5,4
 4    MOD = 2
      GO TO 6
 5    MOD = 3
 6    CONTINUE
      DO 20 I=2,II
20    BET(I)=BETTER(L,I)
      CALL DRAW(NUMPTS,E,BET,    MOD,0,LABEL,ITITLE,.3,0,0,0,0,0,9,15,1,L
     1AST)
 7    CONTINUE
      RETURN
      END
```

THE FOLLOWING SUBROUTINES ARE DEPENDENT UPON A PARTICULAR POTENTIAL SHAPE.

```
    GAUSSIAN POTENTIAL


     SUBROUTINE READY2
     DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
    1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
    2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
    300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
    40),FNORM(100),BETTER(10,100)
     DIMENSION VALUE(40)
     COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
    1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
    2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
    3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
    4TTER
     E(1) = EINT
     DO 199 I = 2,II
     E(I) = E(I-1) + EINC
     FK(I) = SQRTF(E(I)/.41499E-24)
 199 FK2(I)= FK(I) * FK(I)
     LL=2*MANY + 1
     JJ=LL
     FMANY = MANY
     GAUSSIAN DISTRIBUTION
     RA(1) = ROS/AS*(1.- FMANY*CHANGE)
     DO 170 L=1,LL
     S=(1.-2.03*RA(L)+.587*RA(L)**2)/(1.-2.81*RA(L)+1.805*RA(L)**2)
     SS=S-1.
     B=RA(L)*AS/(1.-.641*SS+.568*SS**2-.721*SS**3+1.172*SS**4-1.015*SS*
    1*5)
     POT(L)=S*.229208E-23/B**2
     RN(L)=SQRTF(B**2/2.0604)
 170 RA(L+1) = RA(L) + CHANGE *ROS/AS
     IF(MODE - 1) 190,192,197
 197 PRINT 196
 196 FORMAT (13X,7H ENERGY,5X,12H WAVE NUMBER,3X,20H WAVE NUMBER SQUARE
    1D)
     PRINT 195 ,(E(I), FK(I),FK2(I),I=2,II)
 195 FORMAT (3E20.4)
     PRINT 171
 171 FORMAT(34H EFFECTIVE RANGE/SCATTERING LENGTH)
     PRINT 172,(RA(L) L=1,LL)
 172 FORMAT(E34.10)
     PRINT 194
 194 FORMAT (13X,7H RADIUS)
     PRINT 193, (RN(L), L=1,LL)
 193 FORMAT (E20.4)
     PRINT 1192
1192 FORMAT (16H POTENTIAL DEPTH)
     PRINT 2192,(POT(J),J=1,JJ)
2192 FORMAT(F16.2)
 192 PRINT 191
```

```
191 FORMAT( 34H RADIUS AND ENERGY VALUES COMPUTED)
190 DO 187 I=2,II
    ARGB     = FK(I) * REND
    CALL BESSEL( ARGB,2,VALUE,5, 0)
    VALJ0(I) = VALUE(0)
    VALJ1(I) = VALUE(1)
    VALJ2(I) = VALUE(2)
    CALL BESSEL (ARGB,2,VALUE, 6,0)
    VALN0(I) = VALUE(0)
    VALN1(I) = VALUE(1)
    VALN2(I) = VALUE(2)
    IF(MODE - 1) 187,187,189
189 PRINT 188,I
188 FORMAT(14I5)
187 CONTINUE
    IF(MODE - 1) 184,186,186
186 PRINT 185
185 FORMAT( 26H BESSEL FUNCTIONS COMPUTED)
184 RETURN
    END
```

```
   EXPONENTIAL POTENTIAL


    SUBROUTINE READY2
    DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
   1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
   2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
   300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
   40),FNORM(100),BETTER(10,100)
    DIMENSION VALUE(40)
    COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
   1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
   2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
   3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
   4TTER
    E(1) = EINT
    DO 199 I = 2,II
    E(I) = E(I-1) + EINC
    FK(I) = SQRTF(E(I)/.41499E-24)
199 FK2(I)= FK(I) * FK(I)
    LL=2*MANY + 1
    JJ=LL
    EXPONENTIAL WELL
    FMANY = MANY
    RA(1)=ROS/AS*(1.+FMANY*CHANGE)
    DO 170 L=1,LL
    S=(1.-2.9739*RA(L)+1.3602*RA(L)**2)/(1.-3.7134*RA(L)+3.1177*RA(L)*
   1*2)
    SS=S-1.
    B=RA(L)*AS/(1.-.904*SS+.745*SS**2-.542*SS**3+.174*SS**4)
    POT(L)=S*.751541E-23/B**2
    RN(L)=B/3.5412
170 RA(L+1) = RA(L) - CHANGE *ROS/AS
    IF(MODE - 1) 190,192,197
197 PRINT 196
196 FORMAT (13X,7H ENERGY,5X,12H WAVE NUMBER,3X,20H WAVE NUMBER SQUARE
   1D)
    PRINT 195 ,(E(I), FK(I),FK2(I),I=2,II)
195 FORMAT (3E20.4)
    PRINT 171
171 FORMAT(34H EFFECTIVE RANGE/SCATTERING LENGTH)
    PRINT 172,(RA(L) L=1,LL)
172 FORMAT(E34.10)
    PRINT 194
194 FORMAT (13X,7H RADIUS)
    PRINT 193, (RN(L), L=1,LL)
193 FORMAT (E20.4)
    PRINT 1192
192 FORMAT (16H POTENTIAL DEPTH)
    PRINT 2192,(POT(J),J=1,JJ)
192 FORMAT(F16.2)
192 PRINT 191
```

```
191 FORMAT( 34H RADIUS AND ENERGY VALUES COMPUTED)
190 DO 187 I=2,II
    ARGB      = FK(I) * REND
    CALL BESSEL( ARGB,2,VALUE,5, 0)
    VALJ0(I) = VALUE(0)
    VALJ1(I) = VALUE(1)
    VALJ2(I) = VALUE(2)
    CALL BESSEL (ARGB,2,VALUE, 6,0)
    VALN0(I) = VALUE(0)
    VALN1(I) = VALUE(1)
    VALN2(I) = VALUE(2)
    IF(MODE - 1) 187,187,189
189 PRINT 188,I
188 FORMAT(14I5)
187 CONTINUE
    IF(MODE - 1) 184,186,186
186 PRINT 185
185 FORMAT( 26H BESSEL FUNCTIONS COMPUTED)
184 RETURN
    END
```

YUKAWA POTENTIAL

```
      SUBROUTINE READY2
      DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
     1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
     2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
     300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
     40),FNORM(100),BETTER(10,100)
      DIMENSION VALUE(40)
      COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
     1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
     2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
     3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
     4TTER
      E(1) = EINT
      DO 199 I = 2,II
      E(I) = E(I-1) + EINC
      FK(I) = SQRTF(E(I)/.41499E-24)
  199 FK2(I)= FK(I) * FK(I)
      LL=2*MANY + 1
      JJ=LL
      FMANY = MANY
      RA(1)=ROS/AS*(1.+FMANY*CHANGE)
C     YUKAWA WELL
      DO 170 L=1,LL
      S=(1.-3.5093*RA(L)+2.0354*RA(L)**2)/(1.-4.1348*RA(L)+3.7072*RA(L)*
     1*2)
      SS=S-1.
      B=RA(L)*AS/(1.-1.369*SS+1.093*SS**2-1.127*SS**3+1.005*SS**4-.324*S
     1S**5)
      POT(L)=S*.147585E-23*2.1196/B**2
      RN(L)=B/2.1196
  170 RA(L+1) = RA(L) - CHANGE *ROS/AS
      IF(MODE - 1) 190,192,197
  197 PRINT 196
  196 FORMAT (13X,7H ENERGY,5X,12H WAVE NUMBER,3X,20H WAVE NUMBER SQUARE
     1D)
      PRINT 195 ,(E(I), FK(I),FK2(I),I=2,II)
  195 FORMAT (3E20.4)
      PRINT 171
  171 FORMAT(34H EFFECTIVE RANGE/SCATTERING LENGTH)
      PRINT 172,(RA(L) L=1,LL)
  172 FORMAT(E34.10)
      PRINT 194
  194 FORMAT (13X,7H RADIUS)
      PRINT 193, (RN(L), L=1,LL)
  193 FORMAT (E20.4)
      PRINT 1192
 1192 FORMAT (16H POTENTIAL DEPTH)
      PRINT 2192,(POT(J),J=1,JJ)
 2192 FORMAT(F16.2)
```

```
192 PRINT 191
191 FORMAT( 34H RADIUS AND ENERGY VALUES COMPUTED)
190 DO 187 I=2,II
    ARGB     = FK(I) * REND
    CALL BESSEL( ARGB,2,VALUE,5, 0)
    VALJ0(I) = VALUE(0)
    VALJ1(I) = VALUE(1)
    VALJ2(I) = VALUE(2)
    CALL BESSEL (ARGB,2,VALUE, 6,0)
    VALN0(I) = VALUE(0)
    VALN1(I) = VALUE(1)
    VALN2(I) = VALUE(2)
    IF(MODE - 1) 187,187,189
189 PRINT 188,I
188 FORMAT(14I5)
187 CONTINUE
    IF(MODE - 1) 184,186,186
186 PRINT 185
185 FORMAT( 26H BESSEL FUNCTIONS COMPUTED)
184 RETURN
    END
```

```
   SQUARE WELL POTENTIAL


   SUBROUTINE READY2
   DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
  1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
  2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
  300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
  40),FNORM(100),BETTER(10,100)
   DIMENSION VALUE(40)
   COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
  1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
  2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
  3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
  4TTER
   E(1) = EINT
   DO 199 I = 2,II
   E(I) = E(I-1) + EINC
   FK(I) = SQRTF(E(I)/.41499E-24)
199 FK2(I)= FK(I) * FK(I)
   LL=2*MANY + 1
   JJ=LL
   SQUARE WELL
   FMANY = MANY
   CHANGE SIGN TO - FOR NORMAL OPERATION
   RA(1)=ROS/AS*(1.+FMANY*CHANGE)
   DO 170 L=1,LL
   S=(1.-1.574*RA(L)+.353*RA(L)**2)/(1.-2.384*RA(L)+1.30*RA(L)**2)
   SS=S-1.
   B=RA(L)*AS/(1.-.5*SS+.486*SS**2-.476*SS**3+.378*SS**4-.158*SS**5)
   POT(L) = S*.102276E-23/B**2
   RN(L) =B
170 RA(L+1) = RA(L) - CHANGE *ROS/AS
   IF(MODE - 1) 190,192,197
197 PRINT 196
196 FORMAT (13X,7H ENERGY,5X,12H WAVE NUMBER,3X,20H WAVE NUMBER SQUARE
  1D)
   PRINT 195 ,(E(I), FK(I),FK2(I),I=2,II)
195 FORMAT (3E20.4)
   PRINT 171
171 FORMAT(34H EFFECTIVE RANGE/SCATTERING LENGTH)
   PRINT 172,(RA(L) L=1,LL)
172 FORMAT(E34.10)
   PRINT 194
194 FORMAT (13X,7H RADIUS)
   PRINT 193, (RN(L), L=1,LL)
193 FORMAT (E20.4)
   PRINT 1192
1192 FORMAT (16H POTENTIAL DEPTH)
   PRINT 2192,(POT(J),J=1,JJ)
2192 FORMAT(F16.2)
192 PRINT 191
```

```
191 FORMAT( 34H RADIUS AND ENERGY VALUES COMPUTED)
190 DO 187 I=2,II
    ARGB     = FK(I) * REND
    CALL BESSEL( ARGB,2,VALUE,5, 0)
    VALJ0(I) = VALUE(0)
    VALJ1(I) = VALUE(1)
    VALJ2(I) = VALUE(2)
    CALL BESSEL (ARGB,2,VALUE, 6,0)
    VALN0(I) = VALUE(0)
    VALN1(I) = VALUE(1)
    VALN2(I) = VALUE(2)
    IF(MODE - 1) 187,187,189
189 PRINT 188,I
188 FORMAT(14I5)
187 CONTINUE
    IF(MODE - 1) 184,186,186
186 PRINT 185
185 FORMAT( 26H BESSEL FUNCTIONS COMPUTED)
184 RETURN
    END
```

GAUSSIAN POTENTIAL

```
 SUBROUTINE DERIV (XC,YC,DY)
 DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
40),FNORM(100),BETTER(10,100)
 DIMENSION YC(30), DY(30)
 COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
4TTER
 DY(1) =YC(2)
 DY(2) = - (FK2(K) + V * EXPF(-(XC/A) ** 2)) * YC(1)
 RETURN
 END


 SUBROUTINE DERIV1(XC,YC,DY)
 DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
40),FNORM(100),BETTER(10,100)
 DIMENSION YC(30), DY(30)
 COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
4TTER
 DY(1) = YC(2)
 DY(2) = -(FK2(K) + V * EXPF(-(XC/A)**2) - 2./XC**2)*YC(1)
 RETURN
 END
```

EXPONENTIAL POTENTIAL

```
 SUBROUTINE DERIV (XC,YC,DY)
 DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
40),FNORM(100),BETTER(10,100)
 DIMENSION YC(30), DY(30)
 COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
4TTER
 DY(1) = YC(2)
 DY(2) = - (FK2(K) + V * EXPF(-     XC/A)) * YC(1)
 RETURN
 END


 SUBROUTINE DERIV1(XC,YC,DY)
 DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
40),FNORM(100),BETTER(10,100)
 DIMENSION YC(30), DY(30)
 COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
4TTER
 DY(1) = YC(2)
 DY(2) = - (FK2(K) + V * EXPF(-     XC/A) -2. /XC**2)*YC(1)
 RETURN
 END
```

YUKAWA POTENTIAL

```
 SUBROUTINE DERIV (XC,YC,DY)
 DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
40),FNORM(100),BETTER(10,100)
 DIMENSION YC(30), DY(30)
 COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
4TTER
 DY(1) = YC(2)
 DY(2) = - (FK2(K) + V * (EXPF(-XC/A))/(XC/A))*YC(1)
 RETURN
 END
```

```
 SUBROUTINE DERIV1(XC,YC,DY)
 DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
40),FNORM(100),BETTER(10,100)
 DIMENSION YC(30), DY(30)
 COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
4TTER
 DY(1) = YC(2)
 DY(2) = - (FK2(K) + V * (EXPF(-XC/A))/(XC/A) -2./XC**2) * YC(1)
 RETURN
 END
```

SQUARE WELL POTENTIAL

```
      SUBROUTINE DERIV (XC,YC,DY)
      DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
     1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
     2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
     300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
     40),FNORM(100),BETTER(10,100)
      DIMENSION YC(30), DY(30)
      COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
     1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
     2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
     3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
     4TTER
      DY(1) =YC(2)
      IF(XC -A) 1,1,2
    1 DY(2) = -(FK2(K) + V) * YC(1)
      RETURN
    2 DY(2) =-(FK2(K))* YC(1)
      RETURN
      END


      SUBROUTINE DERIV1(XC,YC,DY)
      DIMENSION E(100),FK(100),FK2(100),RN(100),VALJ0(100),VALJ1(100)
     1,VALJ2(100), VALN0(100), VALN1(100),VALN2(100), R(1000), Y(1000),
     2YDOT(1000), COMPAR0(100), COMPAR1(100), YL(1000),YLDOT(1000),POT(1
     300),DELTA0(100),DELTA1(100),XSEC0(100),XSEC1(100),TOTAL(100),RA(10
     40),FNORM(100),BETTER(10,100)
      DIMENSION YC(30), DY(30)
      COMMON E,EINT,EINC, FK, FK2, RINT, RINC, RN, A, II, LL, NN, MODE,
     1REND,VALJ0,VALJ1,VALJ2,VALN0,VALN1,VALN2,JJ,M, NUMBER, R,I,L,V,K,
     2N,Y, YDOT, YL, YLDOT, III, COMPAR0, COMPAR1,POT,VINT,VINC,MOMENTA,
     3DELTA0,DELTA1,XSEC0,XSEC1,TOTAL,RA,ROS,AS,CHANGE,MANY,S,B,FNORM,BE
     4TTER
      DY(1) =YC(2)
      IF(XC -A) 1,1,2
    1 DY(2) = -(FK2(K) + V - 2./XC**2)*YC(1)
      RETURN
    2 DY(2) = -(FK2(K) - 2./ XC**2) *YC(1)
      RETURN
      END
```

THE FOLLOWING DEFINITIONS FOR B MUST BE SUBSTITUTED IN READY2 FOR VARYIN CATTERING LENGTH. THE DEFINITION PROVIDED IN READY2 IN THE LISTING IS FOR ARIATIONS IN EFFECTIVE RANGE.

YUKAWA WELL

```
B=ROS/(1.-1.369*SS+1.093*SS**2-1.127*SS**3+1.005*SS**4-.324*SS**5)
```

SQUARE WELL

```
B=ROS/(1.-.5*(S-1.)+.486*(S-1.)**2-.476*(S-1.)**3+.378*(S-1.)**4
1-.158*(S-1.)**5)
```

EXPONENTIAL WELL

```
B=ROS/(1.-.904*SS+.745*SS**2-.543*SS**3+.174*SS**4)
```

GAUSSIAN WELL

```
B=ROS/(1.-.641*SS+.568*SS**2-.721*SS**3+1.172*SS**4-1.015*SS**5)
```

APPENDIX IV

SAMPLE RESULTS

These results are for a yukawa potential with $r_{o,s}^*$ fixed at 2.46 Fm and a_s varying in I 16% deviation from $a_s^* = 23.677$ FM.

```
EFFECTIVE RANGE/SCATTERING LENGTH
                   -.8727457026E-01
               RADIUS      POTENTIAL DEPTH
            .1086E-12             .5621E+02
ENERGY PHASE ANGLE CROSS SECTION,
             L=0          L=0

   .10     .9113E+00        .3257E-22
   .20     .1037E+01        .1933E-22
   .30     .1094E+01        .1372E-22
   .40     .1125E+01        .1062E-22
   .50     .1145E+01        .8649E-23
   .60     .1158E+01        .7290E-23
   .70     .1166E+01        .6295E-23
   .80     .1172E+01        .5535E-23
   .90     .1176E+01        .4936E-23
  1.00     .1178E+01        .4451E-23
  1.10     .1180E+01        .4051E-23
  1.20     .1180E+01        .3716E-23
  1.30     .1180E+01        .3430E-23
  1.40     .1180E+01        .3184E-23
  1.50     .1179E+01        .2970E-23
  1.60     .1178E+01        .2781E-23
  1.70     .1176E+01        .2615E-23
  1.80     .1175E+01        .2466E-23
  1.90     .1173E+01        .2333E-23
  2.00     .1172E+01        .2213E-23
  2.10     .1170E+01        .2105E-23
  2.20     .1168E+01        .2006E-23
  2.30     .1166E+01        .1915E-23
  2.40     .1164E+01        .1832E-23
  2.50     .1161E+01        .1755E-23
  2.60     .1159E+01        .1685E-23
ENERGY          NORMALIZED CROSS SECTION
   .10            .11366967E+01
   .20            .10869311E+01
   .30            .10658047E+01
   .40            .10541020E+01
   .50            .10466564E+01
   .60            .10414947E+01
   .70            .10377002E+01
   .80            .10347892E+01
   .90            .10324821E+01
  1.00            .10306062E+01
  1.10            .10290489E+01
  1.20            .10277338E+01
  1.30            .10266071E+01
  1.40            .10256300E+01
  1.50            .10247736E+01
  1.60            .10240161E+01
  1.70            .10233405E+01
  1.80            .10227338E+01
  1.90            .10221853E+01
  2.00            .10216868E+01
  2.10            .10212311E+01
  2.20            .10208129E+01
  2.30            .10204272E+01
  2.40            .10200702E+01
  2.50            .10197386E+01
  2.60            .10194296E+01
```

```
EFFECTIVE RANGE/SCATTERING LENGTH
                 -.9143050217E-01
             RADIUS    POTENTIAL DEPTH
          .1083E-12            .5641E+02
ENERGY PHASE ANGLE CROSS SECTION,
           L=0        L=0

   .10   .8913E+00      .3155E-22
   .20   .1020E+01      .1894E-22
   .30   .1079E+01      .1351E-22
   .40   .1112E+01      .1048E-22
   .50   .1133E+01      .8553E-23
   .60   .1146E+01      .7218E-23
   .70   .1156E+01      .6238E-23
   .80   .1162E+01      .5489E-23
   .90   .1166E+01      .4897E-23
  1.00   .1169E+01      .4419E-23
  1.10   .1171E+01      .4023E-23
  1.20   .1172E+01      .3691E-23
  1.30   .1172E+01      .3408E-23
  1.40   .1172E+01      .3164E-23
  1.50   .1172E+01      .2952E-23
  1.60   .1171E+01      .2765E-23
  1.70   .1170E+01      .2600E-23
  1.80   .1168E+01      .2453E-23
  1.90   .1167E+01      .2321E-23
  2.00   .1165E+01      .2202E-23
  2.10   .1164E+01      .2094E-23
  2.20   .1162E+01      .1995E-23
  2.30   .1160E+01      .1906E-23
  2.40   .1158E+01      .1823E-23
  2.50   .1156E+01      .1747E-23
  2.60   .1154E+01      .1677E-23
ENERGY      NORMALIZED CROSS SECTION
   .10       .11011434E+01
   .20       .10649739E+01
   .30       .10493686E+01
   .40       .10406591E+01
   .50       .10350936E+01
   .60       .10312243E+01
   .70       .10283743E+01
   .80       .10261847E+01
   .90       .10244473E+01
  1.00       .10230335E+01
  1.10       .10218590E+01
  1.20       .10208666E+01
  1.30       .10200161E+01
  1.40       .10192782E+01
  1.50       .10186313E+01
  1.60       .10180589E+01
  1.70       .10175484E+01
  1.80       .10170898E+01
  1.90       .10166753E+01
  2.00       .10162984E+01
  2.10       .10159539E+01
  2.20       .10156377E+01
  2.30       .10153461E+01
  2.40       .10150762E+01
  2.50       .10148255E+01
  2.60       .10145919E+01
```

```
EFFECTIVE RANGE/SCATTERING LENGTH
                  -.9558643409E-01
             RADIUS      POTENTIAL DEPTH
          .1080E-12              .5661E+02
ENERGY PHASE ANGLE CROSS SECTION,
            L=0           L=0

   .10     .8719E+00        .3056E-22
   .20     .1004E+01        .1855E-22
   .30     .1064E+01        .1329E-22
   .40     .1099E+01        .1034E-22
   .50     .1121E+01        .8457E-23
   .60     .1135E+01        .7145E-23
   .70     .1145E+01        .6181E-23
   .80     .1152E+01        .5443E-23
   .90     .1157E+01        .4859E-23
  1.00     .1161E+01        .4386E-23
  1.10     .1163E+01        .3994E-23
  1.20     .1164E+01        .3666E-23
  1.30     .1165E+01        .3386E-23
  1.40     .1165E+01        .3144E-23
  1.50     .1165E+01        .2934E-23
  1.60     .1164E+01        .2749E-23
  1.70     .1163E+01        .2585E-23
  1.80     .1162E+01        .2439E-23
  1.90     .1161E+01        .2308E-23
  2.00     .1159E+01        .2190E-23
  2.10     .1158E+01        .2083E-23
  2.20     .1156E+01        .1985E-23
  2.30     .1154E+01        .1896E-23
  2.40     .1152E+01        .1814E-23
  2.50     .1150E+01        .1738E-23
  2.60     .1148E+01        .1669E-23
ENERGY          NORMALIZED CROSS SECTION
   .10             .10664810E+01
   .20             .10431380E+01
   .30             .10329030E+01
   .40             .10271472E+01
   .50             .10234527E+01
   .60             .10208768E+01
   .70             .10189756E+01
   .80             .10175126E+01
   .90             .10163506E+01
  1.00             .10154041E+01
  1.10             .10146173E+01
  1.20             .10139521E+01
  1.30             .10133817E+01
  1.40             .10128867E+01
  1.50             .10124526E+01
  1.60             .10120684E+01
  1.70             .10117257E+01
  1.80             .10114178E+01
  1.90             .10111394E+01
  2.00             .10108863E+01
  2.10             .10106549E+01
  2.20             .10104425E+01
  2.30             .10102467E+01
  2.40             .10100654E+01
  2.50             .10098970E+01
  2.60             .10097400E+01
```

```
EFFECTIVE RANGE/SCATTERING LENGTH
                 -.9974236601E-01
             RADIUS     POTENTIAL DEPTH
           .1077E-12           .5680E+02
ENERGY PHASE ANGLE CROSS SECTION,
           L=0          L=0

   .10   .8531E+00      .2959E-22
   .20   .9875E+00      .1817E-22
   .30   .1050E+01      .1308E-22
   .40   .1086E+01      .1021E-22
   .50   .1109E+01      .8360E-23
   .60   .1125E+01      .7073E-23
   .70   .1135E+01      .6124E-23
   .80   .1143E+01      .5396E-23
   .90   .1148E+01      .4820E-23
  1.00   .1152E+01      .4352E-23
  1.10   .1155E+01      .3966E-23
  1.20   .1156E+01      .3641E-23
  1.30   .1157E+01      .3363E-23
  1.40   .1158E+01      .3124E-23
  1.50   .1157E+01      .2916E-23
  1.60   .1157E+01      .2733E-23
  1.70   .1156E+01      .2570E-23
  1.80   .1155E+01      .2425E-23
  1.90   .1154E+01      .2296E-23
  2.00   .1153E+01      .2178E-23
  2.10   .1152E+01      .2072E-23
  2.20   .1150E+01      .1975E-23
  2.30   .1148E+01      .1886E-23
  2.40   .1147E+01      .1805E-23
  2.50   .1145E+01      .1730E-23
  2.60   .1143E+01      .1661E-23
ENERGY       NORMALIZED CROSS SECTION
   .10        .10327555E+01
   .20        .10214671E+01
   .30        .10164376E+01
   .40        .10135874E+01
   .50        .10117498E+01
   .60        .10104647E+01
   .70        .10095142E+01
   .80        .10087817E+01
   .90        .10081992E+01
  1.00        .10077243E+01
  1.10        .10073292E+01
  1.20        .10069950E+01
  1.30        .10067083E+01
  1.40        .10064594E+01
  1.50        .10062411E+01
  1.60        .10060478E+01
  1.70        .10058753E+01
  1.80        .10057203E+01
  1.90        .10055802E+01
  2.00        .10054527E+01
  2.10        .10053363E+01
  2.20        .10052293E+01
  2.30        .10051307E+01
  2.40        .10050394E+01
  2.50        .10049546E+01
  2.60        .10048756E+01
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EFFECTIVE RANGE/SCATTERING LENGTH
                     -.1038982979E+00
           RADIUS        POTENTIAL DEPTH
         .1074E-12                .5700E+02
ENERGY PHASE ANGLE CROSS SECTION,
            L=0            L=0

   .10    .8350E+00          .2866E-22
   .20    .9717E+00          .1778E-22
   .30    .1036E+01          .1287E-22
   .40    .1074E+01          .1007E-22
   .50    .1098E+01          .8263E-23
   .60    .1114E+01          .6999E-23
   .70    .1125E+01          .6066E-23
   .80    .1133E+01          .5349E-23
   .90    .1139E+01          .4781E-23
  1.00    .1143E+01          .4319E-23
  1.10    .1146E+01          .3937E-23
  1.20    .1148E+01          .3615E-23
  1.30    .1150E+01          .3341E-23
  1.40    .1150E+01          .3104E-23
  1.50    .1150E+01          .2898E-23
  1.60    .1150E+01          .2716E-23
  1.70    .1150E+01          .2555E-23
  1.80    .1149E+01          .2412E-23
  1.90    .1148E+01          .2283E-23
  2.00    .1147E+01          .2166E-23
  2.10    .1146E+01          .2061E-23
  2.20    .1144E+01          .1965E-23
  2.30    .1143E+01          .1877E-23
  2.40    .1141E+01          .1796E-23
  2.50    .1140E+01          .1721E-23
  2.60    .1138E+01          .1653E-23
ENERGY        NORMALIZED CROSS SECTION
   .10          .10000000E+01
   .20          .10000000E+01
   .30          .10000000E+01
   .40          .10000000E+01
   .50          .10000000E+01
   .60          .10000000E+01
   .70          .10000000E+01
   .80          .10000000E+01
   .90          .10000000E+01
  1.00          .10000000E+01
  1.10          .10000000E+01
  1.20          .10000000E+01
  1.30          .10000000E+01
  1.40          .10000000E+01
  1.50          .10000000E+01
  1.60          .10000000E+01
  1.70          .10000000E+01
  1.80          .10000000E+01
  1.90          .10000000E+01
  2.00          .10000000E+01
  2.10          .10000000E+01
  2.20          .10000000E+01
  2.30          .10000000E+01
  2.40          .10000000E+01
  2.50          .10000000E+01
  2.60          .10000000E+01
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EFFECTIVE RANGE/SCATTERING LENGTH
                 -.1080542298E+00
             RADIUS     POTENTIAL DEPTH
          .1071E-12           .5719E+02
ENERGY PHASE ANGLE CROSS SECTION,
           L=0        L=0

   .10   .8175E+00      .2775E-22
   .20   .9562E+00      .1741E-22
   .30   .1022E+01      .1266E-22
   .40   .1061E+01      .9934E-23
   .50   .1086E+01      .8166E-23
   .60   .1103E+01      .6926E-23
   .70   .1115E+01      .6008E-23
   .80   .1124E+01      .5302E-23
   .90   .1130E+01      .4741E-23
  1.00   .1135E+01      .4286E-23
  1.10   .1138E+01      .3908E-23
  1.20   .1141E+01      .3590E-23
  1.30   .1142E+01      .3318E-23
  1.40   .1143E+01      .3084E-23
  1.50   .1144E+01      .2880E-23
  1.60   .1144E+01      .2700E-23
  1.70   .1143E+01      .2540E-23
  1.80   .1143E+01      .2398E-23
  1.90   .1142E+01      .2270E-23
  2.00   .1141E+01      .2155E-23
  2.10   .1140E+01      .2050E-23
  2.20   .1139E+01      .1954E-23
  2.30   .1137E+01      .1867E-23
  2.40   .1136E+01      .1787E-23
  2.50   .1134E+01      .1713E-23
  2.60   .1133E+01      .1645E-23
ENERGY      NORMALIZED CROSS SECTION
   .10       .96823662E+00
   .20       .97877150E+00
   .30       .98361602E+00
   .40       .98640398E+00
   .50       .98821811E+00
   .60       .98949447E+00
   .70       .99044257E+00
   .80       .99117554E+00
   .90       .99175986E+00
  1.00       .99223716E+00
  1.10       .99263482E+00
  1.20       .99297162E+00
  1.30       .99326084E+00
  1.40       .99351216E+00
  1.50       .99373277E+00
  1.60       .99392818E+00
  1.70       .99410261E+00
  1.80       .99425941E+00
  1.90       .99440125E+00
  2.00       .99453026E+00
  2.10       .99464820E+00
  2.20       .99475652E+00
  2.30       .99485641E+00
  2.40       .99494888E+00
  2.50       .99503479E+00
  2.60       .99511485E+00
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EFFECTIVE RANGE/SCATTERING LENGTH
                 -.1122101618E+00
            RADIUS      POTENTIAL DEPTH
           .1068E-12              .5738E+02
ENERGY PHASE ANGLE CROSS SECTION,
           L=0        L=0

   .10   .8005E+00      .2686E-22
   .20   .9412E+00      .1703E-22
   .30   .1009E+01      .1245E-22
   .40   .1049E+01      .9797E-23
   .50   .1075E+01      .8068E-23
   .60   .1093E+01      .6852E-23
   .70   .1105E+01      .5950E-23
   .80   .1115E+01      .5254E-23
   .90   .1122E+01      .4702E-23
  1.00   .1127E+01      .4252E-23
  1.10   .1130E+01      .3879E-23
  1.20   .1133E+01      .3564E-23
  1.30   .1135E+01      .3296E-23
  1.40   .1136E+01      .3064E-23
  1.50   .1137E+01      .2861E-23
  1.60   .1137E+01      .2683E-23
  1.70   .1137E+01      .2525E-23
  1.80   .1136E+01      .2384E-23
  1.90   .1136E+01      .2257E-23
  2.00   .1135E+01      .2143E-23
  2.10   .1134E+01      .2039E-23
  2.20   .1133E+01      .1944E-23
  2.30   .1132E+01      .1857E-23
  2.40   .1130E+01      .1778E-23
  2.50   .1129E+01      .1704E-23
  2.60   .1127E+01      .1636E-23
ENERGY      NORMALIZED CROSS SECTION
   .10       .93747776E+00
   .20       .95781212E+00
   .30       .96730933E+00
   .40       .97281728E+00
   .50       .97641799E+00
   .60       .97895922E+00
   .70       .98085103E+00
   .80       .98231599E+00
   .90       .98348532E+00
  1.00       .98444143E+00
  1.10       .98523866E+00
  1.20       .98591430E+00
  1.30       .98649480E+00
  1.40       .98699944E+00
  1.50       .98744260E+00
  1.60       .98783524E+00
  1.70       .98818582E+00
  1.80       .98850104E+00
  1.90       .98878622E+00
  2.00       .98904565E+00
  2.10       .98928285E+00
  2.20       .98950072E+00
  2.30       .98970165E+00
  2.40       .98988768E+00
  2.50       .99006050E+00
  2.60       .99022157E+00
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 EFFECTIVE RANGE/SCATTERING LENGTH
                   -.1163660937E+00
           RADIUS       POTENTIAL DEPTH
        .1065E-12                .5757E+02
 ENERGY PHASE ANGLE CROSS SECTION,
            L=0           L=0

    .10    .7842E+00         .2601E-22
    .20    .9264E+00         .1667E-22
    .30    .9957E+00         .1224E-22
    .40    .1037E+01         .9661E-23
    .50    .1064E+01         .7971E-23
    .60    .1082E+01         .6778E-23
    .70    .1096E+01         .5892E-23
    .80    .1106E+01         .5207E-23
    .90    .1113E+01         .4662E-23
   1.00    .1118E+01         .4218E-23
   1.10    .1122E+01         .3850E-23
   1.20    .1125E+01         .3539E-23
   1.30    .1127E+01         .3273E-23
   1.40    .1129E+01         .3044E-23
   1.50    .1130E+01         .2843E-23
   1.60    .1130E+01         .2667E-23
   1.70    .1130E+01         .2510E-23
   1.80    .1130E+01         .2370E-23
   1.90    .1130E+01         .2244E-23
   2.00    .1129E+01         .2131E-23
   2.10    .1128E+01         .2028E-23
   2.20    .1127E+01         .1934E-23
   2.30    .1126E+01         .1848E-23
   2.40    .1125E+01         .1769E-23
   2.50    .1124E+01         .1696E-23
   2.60    .1122E+01         .1628E-23
 ENERGY        NORMALIZED CROSS SECTION
    .10          .90772760E+00
    .20          .93714849E+00
    .30          .95110169E+00
    .40          .95925670E+00
    .50          .96461287E+00
    .60          .96840489E+00
    .70          .97123418E+00
    .80          .97342875E+00
    .90          .97518273E+00
   1.00          .97661832E+00
   1.10          .97781634E+00
   1.20          .97883232E+00
   1.30          .97970572E+00
   1.40          .98046532E+00
   1.50          .98113263E+00
   1.60          .98172405E+00
   1.70          .98225227E+00
   1.80          .98272732E+00
   1.90          .98315716E+00
   2.00          .98354827E+00
   2.10          .98390590E+00
   2.20          .98423442E+00
   2.30          .98453743E+00
   2.40          .98481798E+00
   2.50          .98507864E+00
   2.60          .98532158E+00
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EFFECTIVE RANGE/SCATTERING LENGTH
                  -.1205220256E+00
          RADIUS      POTENTIAL DEPTH
        .1062E-12              .5776E+02
ENERGY PHASE ANGLE CROSS SECTION,
          L=0           L=0

   .10    .7684E+00         .2519E-22
   .20    .9120E+00         .1630E-22
   .30    .9828E+00         .1203E-22
   .40    .1025E+01         .9525E-23
   .50    .1053E+01         .7873E-23
   .60    .1072E+01         .6704E-23
   .70    .1086E+01         .5833E-23
   .80    .1097E+01         .5159E-23
   .90    .1104E+01         .4622E-23
  1.00    .1110E+01         .4184E-23
  1.10    .1114E+01         .3820E-23
  1.20    .1118E+01         .3513E-23
  1.30    .1120E+01         .3250E-23
  1.40    .1122E+01         .3023E-23
  1.50    .1123E+01         .2825E-23
  1.60    .1124E+01         .2650E-23
  1.70    .1124E+01         .2495E-23
  1.80    .1124E+01         .2356E-23
  1.90    .1124E+01         .2231E-23
  2.00    .1123E+01         .2119E-23
  2.10    .1122E+01         .2017E-23
  2.20    .1122E+01         .1923E-23
  2.30    .1121E+01         .1838E-23
  2.40    .1119E+01         .1760E-23
  2.50    .1118E+01         .1687E-23
  2.60    .1117E+01         .1620E-23
ENERGY          NORMALIZED CROSS SECTION
   .10             .87898331E+00
   .20             .91680363E+00
   .30             .93501300E+00
   .40             .94573795E+00
   .50             .95281525E+00
   .60             .95784165E+00
   .70             .96160045E+00
   .80             .96452096E+00
   .90             .96685819E+00
  1.00             .96877317E+00
  1.10             .97037256E+00
  1.20             .97172985E+00
  1.30             .97289732E+00
  1.40             .97391316E+00
  1.50             .97480592E+00
  1.60             .97559741E+00
  1.70             .97630453E+00
  1.80             .97694060E+00
  1.90             .97751627E+00
  2.00             .97804014E+00
  2.10             .97851924E+00
  2.20             .97895939E+00
  2.30             .97936541E+00
  2.40             .97974136E+00
  2.50             .98009068E+00
  2.60             .98041627E+00
```

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